THE USE OF THE JOINT GENERALIZED LEAST SQUARES ESTIMATION TECHNIQUE IN THE AGGREGATION OF SUBSYSTEM COST ESTIMATING RELATIONSHIPS FOR SHIPS

Arthur William Newlon Jr.

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THESIS

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by

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ABSTRACT

This study addresses the problem of uncertainty in the aggregation of subsystem cost estimating relationships (CERs) for destroyer type naval ships. The case of correlations between subsystems is discussed with respect to total cost estimates, total cost variance, and prediction intervals. The joint generalized least squares method (JGLS) is developed and applied to the ship cost estimating problem. The models analyzed were developed by D. M. Hernon and R. R. McCumber, Estimation of Destroyer Type Naval Ship Procurement Costs, utilizing the data base from Resource Management Corporation Report CR-058, the Navships Cost Model. One model utilizes 9 subsystem CERs to obtain a total cost estimate, while the second model utilizes 4 subsystem CERs. Three solution methods were used for purposes of comparison; the JGLS method assuming correlated subsystems, the least squares method assuming correlated subsystems, and the least squares method assuming independent subsystems. The JGLS method is shown to provide the most meaningful results for correlated subsystems. A computer program and user's guide are provided to conduct JGLS analysis.

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I. INTRODUCTION

Due to cost overruns, Congressional concern, and a continuing need for better planning estimates, it is imperative that new techniques be developed and old techniques refined in order to obtain better estimates for major weapon system procurement and construction. Along with these techniques a better understanding of the factors and forces that determine cost is required.

The area of concern in this paper is with Naval ship procurement and construction costs. The cost of a ship is a result of many factors and forces all of which it would be impossible to define and measure. Thus this cost is subject to variation. When estimating the cost of a ship it is necessary to provide both the estimate itself and as clear a picture as possible of the variability surrounding the estimate.

The general objective of this paper is to analyze the variability of ship cost estimates with respect to the Joint Generalized Least Squares (JGLS) estimation technique, and the Least Squares (LS) regression estimation technique. It is hypothesized that the Joint Generalized Least Squares Estimation technique takes advantage of additional information available and allows a more meaningful statement of the variance of a ship cost estimate.

A. NAVAL SHIP PROCUREMENT AND CONSTRUCTION COSTS

Naval ships may be thought of as major weapon systems consisting of numerous subsystems including:

Propulsion
Hull
Weapons and fire control systems
Auxiliary

There are two basic methods used to predict ships costs. One method is to estimate the construction costs for each subsystem, which are then summed to obtain total ship cost, [Ref. 4 and 6]. The other method is to estimate total ship cost by using the performance and operational characteristics such as, speed or number of missile launchers as explanatory variables, [Ref. 2-3].

The first method is the one of concern in this paper. Here the subsystem costs are estimated by using the physical and performance characteristics of the ship as explanatory variables, normally utilizing the Standard Linear Least Squares multiple regression technique. Weight is often used as a primary explanatory variable in addition to many other characteristics. In order to make meaningful statements about the variability of the total cost of the ship some aggregation technique must be employed on the individual subsystem relations. The aggregation technique used depends upon the assumptions concerning the correlation of disturbances that exist between the subsystem relations. If the assumption of independence between subsystem relations can

be made (a very restrictive assumption) straightforward linear combinations may be used to estimate total cost variance and prediction intervals; call this method LS (independence). This would seem a highly suspect assumption due to the interdependencies of subsystems aboard ships. A more reasonable assumption would be that there are correlations between subsystems. If these interdependencies are recognized, methods do exist to make more meaningful statements about the uncertainty surrounding total cost. An extension of the least squares method may be made by constructing an estimated covariance matrix of the least squares estimates, assuming correlated disturbances; call this method LS (correlated). Finally, if correlations between subsystems are assumed, the joint generalized least squares technique allows this information to be used in determining a set of joint estimates and in constructing an estimated covariance matrix of the joint estimates. This assumption is less restrictive than the independence assumption and in fact should provide more meaningful results; call this method JGLS. Discussion and development of these three methods is contained in Section II, Nature of the Problem and Section III, Methodology of Joint Generalized Least Squares Estimation.

II. NATURE OF THE PROBLEM

A. RECENT SHIP COST ESTIMATING MODELS AND THE TREATMENT OF UNCERTAINTY IN AGGREGATE PREDICTIONS.

1. Resource Management Corporation Report CR-058 to NAVSHIPS

Resource Management Corporation derived a statistical cost model for NAVSHIPS in which ship construction costs are estimated by subsystem and then aggregating these costs to arrive at Basic Contract Cost, and Total Ship End cost. The following subsystem breakdown was used:

1	Hull	6	Outfitting
2	Propulsion	7	Armament
3	Electrical	8	Design and Engineering
4	Communication and Control	9	Construction Services
5	Auxiliary		

The summation of these nine cost categories plus profit and overhead was defined as basic contract cost and formed the nine-subsystem cost model. RMC also developed a condensed four-subsystem cost model composed of the following categories:

1	Hull Group	:	Hull, Design and Engineering, Construction Services
2	Propulsion Group	:	Propulsion, Auxiliary
3	Armament Group	:	Armament, Electrical, Communication and Control
4	Outfitting Group	:	Outfitting

Again, the summation of these four cost categories plus profit and overhead was defined as basic contract cost.

In determining total end costs RMC used NAVSHIPS records of the Shipbuilding and Conversion, Navy (SCN) fund. NAVSHIPS hardcore cost was subsequently defined as basic contract cost plus miscellaneous end costs and electronics end costs. The total end cost then became NAVSHIPS hardcore cost plus weapons end cost. CERs were developed from the data base for predicting basic contract cost and miscellaneous end cost. No CERs were developed for electronics or weapons end costs.

The models were developed for six different classes of ships:

1. Aircraft Carriers
2. Destroyer
3. Submarine
4. Auxiliary
5. Amphibious
6. Patrol/minesweeping

The area of interest in the Hernon and McCumber paper, which will be described next, and also in this paper is with the destroyer models developed. Reference 6 gives the full details of the RMC study and results.

Taking now a point of departure from the general RMC study development, their treatment of uncertainty will be discussed. The treatment of uncertainty involves basically two steps. The first step is to develop the best set of CERs from the data available based on a regression strategy which outlines the basic statistical properties to be used as criteria in the choice. The second step is to analyze and report as thoroughly as possible the variance which surrounds

the CERs developed and, particularly, their aggregation to obtain total cost.

Two points arise from the RMC study with respect to the treatment of uncertainty. In the first, RMC uses the coefficient of determination, R^2, as a measure of the goodness of fit of the regression equation to the data, where

$$R^2 = \frac{\sum_i (\hat{Y}_i - \bar{Y})^2}{\sum_i (Y_i - \bar{Y})^2}$$

The ratio of the sum of squares explained to the total sum of squares of Y adjusted for the mean.

This statistic tends to overstate the goodness of fit as it does not take into consideration the degrees of freedom. A more acceptable statistic would have been the adjusted coefficient of determination (adjusted for degrees of freedom) where R^2_{ADJ} is defined below [see Ref. 7]

$$R^2_{ADJ} = \frac{\frac{1}{n-k} \cdot \Sigma\, e_\alpha^2}{\frac{1}{n-1} \Sigma\, (Y_\alpha - \bar{Y})^2} - 1$$

where $e_\alpha = (\hat{Y}_\alpha - Y_\alpha)$

α is the observation

The second point concerning the reporting of total cost variance deals with prediction intervals on predicted total cost. The RMC study implicitly assumes that the errors in the estimates of costs for the individual subsystems are independent of one another (non-correlated), though they

fail to state this assumption directly. This seems highly unlikely since one shipyard produces several subsystems for a ship and the factors that contribute to the errors in the estimates of one subsystem may very well be the same or partially the same as those that contribute to the errors in the estimates of another subsystem. The allocation of overhead is one example. The factors that contribute to the errors in estimating costs for the propulsion and auxiliary subsystems might very well be the same since these subsystems are in actuality very much dependent upon one another in the ship itself.

Though it is not reported directly in the RMC study, Total cost variance is treated as the summation of the variance not explained in the individual subsystem CERs.

$$V = \sum_{i=1}^{L} S_i^2$$

where V is total cost

S_i^2 is the variance of subsystem i

L is the number of subsystems in the model

Prediction intervals on total cost are reported in the RMC study as the summation of the individual subsystem prediction intervals, where predicted cost is the summation of the subsystem predicted costs.

$$\sum_{i=1}^{L} \hat{c}_i \pm t_{\alpha/2} \sqrt{\sum_{i=1}^{L} A_i}$$

where $\hat{c}_i$ = predicted cost for subsystem i

$A_i = S_i^2(1 + X_0'(X'X)^{-1}X_0)$

the contribution for the ith subsystem

Acting as if the subsystems are independent may lead to substantial overstatements of the confidence one should have in any given total cost estimate as this paper will endeavor to show.

2. Estimation of Destroyer Type Naval Ship Procurement Costs, by D. M. Hernon and R. R. McCumber Jr.

This paper is essentially an extension of the study direction of the RMC study. The objective as stated was,

> "... To develop a model for the prediction of total procurement cost of destroyer type naval ships that increases in precision as input data is refined and, hopefully, approaches a level of quality acceptable in cost-effectiveness studies and eventually for fiscal planning purposes."

Their approach was to first correct numerous deficiencies noted in the RMC study. Because their concern was the prediction of total cost rather than the identification of basic contract costs and separate end cost, data was aggregated with the basic contract cost data as follows:

a. Electronic end cost was added to command and control cost
b. Weapons end cost was added to armament cost
c. Miscellaneous end cost was added to construction services cost.

Next they examined three models; a nine-subsystem cost model, a four-subsystem cost model and finally a single cost estimation equation. The final step was to use the models developed to estimate the total procurement cost of a destroyer type ship under development and compare the predictions to the best available NAVSHIPS estimate.

The study used as a primary criteria for comparing the prediction values of these models the estimate of total cost variance associated with each model. They used two methods to estimate total cost variance. The first method was called the summation method. This is the same as the treatment used by RMC in their study, i.e., the summation of the variance not explained in the individual subsystem CERs. Again, this method requires the assumption that the errors of the individual CERs are independent of one another. Hernon and McCumber acknowledge the difficulties with this assumption and use this method to obtain a minimum total cost variance that may be attained by utilizing that particular set of model CERs.

The second method of total cost variance estimation involves the calculation of a total cost mean square residual value (MSR) for each model. The following method was used to calculate this value:

$$MSR = \frac{\sum_{i=1}^{N} \left[\sum_{j=1}^{L} (\text{Residual}_{ij}) \right]^2}{N - M - L}$$

where N is the number of ships

M is the number of variables utilized in all CERs of the model

L is the number of CERs utilized in the model

Essentially, the residual values produced for each CER for a given ship are summed producing an aggregate of the individual CER residual values. The total cost residuals are squared and summed for all observations (ships). If this quantity is then corrected for degrees of freedom, an estimate of variance is produced for a given model. Hernon and McCumber considered the MSR method to produce an upper bound on the total cost variance estimate, a value below which the estimate of total cost variance is expected to lie.

Hernon and McCumber did not report prediction intervals in their paper. Their treatment of total cost variance did not permit utilization of the standard linear least squares methodology for handling prediction intervals. The coefficient of determination which they reported was the same as the one which the RMC study used and is subject to the comments made earlier concerning it.

3. RAND Report, "Confidence in Estimated Airframe Costs: Assessment in Aggregate Predictions", F.S. Timson and D.P. Tihansky

This report addresses the general problem of confidence measures for multi-equation prediction models utilizing in the development specific results from airframe cost estimation. The report describes how prediction intervals can

be calculated for sums of component regressions. The regressions can be treated as independent and either Student t or normal statistics used (the same treatment as the RMC study), or the regressions can be treated as correlated and normal statistics used.

A simple method permits the determination of the degree of correlation between the individual regressions. It is assumed that there exists a correlated normal distribution for total costs. Consider two regressions derived from a common data base consisting of the same number of observations. Assume the error terms of the two regressions are distributed bivariate normal with zero mean then

$$(\varepsilon_{i1}, \varepsilon_{i2}) \sim N(0,\Omega)$$

$$\Omega = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$$

where i refers to the observation number and Ω is the variance-covariance matrix.

Then if ρ is the correlation between ε_{i1} and ε_{i2} over all observations $\rho = \dfrac{\sigma_{12}}{\sigma_1 \sigma_2}$. ρ may be estimated from the sample as

$$\hat{\rho}_{12} = \frac{\sum_i (Y_{i1} - \hat{Y}_{i1})(Y_{i2} - \hat{Y}_{i2})}{\sqrt{\sum_i (Y_{i1} - \hat{Y}_{i1})^2 \sum_i (Y_{i2} - \hat{Y}_{i2})^2}}$$

When cost models are significantly correlated a method for treating the situation is described. This method assumes normality as a reasonably good approximation to the actual probability distribution. The true variance, W, of the sum of normal costs is expressed as

$$W = \sum_i \sum_j \sigma_{ij} \left(1 + X'_0 (X'X)^{-1} X_0\right)$$

where X_0 is a specified set of values of the independent variables

X is an observed n x K matrix of rank K consisting of values taken by K explanatory variables

σ_{ij} is the covariance between models i and j.
An approximation for which is

$$s_{ij} = \frac{\sum_{q=1}^{N} e_{qi}\, e_{qj}}{N - K - 1}$$

the sums of products of the residuals of models i and j over N observations for observation q.

Prediction interval bounds for the aggregate cost are thus approximated at some confidence level α by

$$\hat{C}_0 \pm N(\alpha)\sqrt{\sum_i \sum_j s_{ij}\left(1 + X_0'(X'X)^{-1} X_0\right)}$$

where $\hat{C}_0$ the predicted total cost, is the sum of the individual predicted costs of the subsystems.

In comparing prediction interval widths for the airframe cost model for independent normal and correlated normal assumptions at a fixed level of confidence, correlated normal distributions generally give the widest intervals.

It should be noted that in this formulation each subsystem CER uses exactly the same explanatory variables putting a very limiting constraint on the use of this method in multi-equation prediction models. In Naval ship cost models it is highly probable that there do exist correlations between subsystem CERs. The CERs developed in the RMC study and by Hernon and McCumber involve different explanatory variables in the subsystem CERs, hence the above described technique may not be applied. The Joint Generalized Least Squares Estimation Technique is a method which will produce more accurate results than the least squares (independence) technique under the circumstances of correlated residuals and different explanatory variables.

B. THESIS OBJECTIVES

The objectives of the thesis are twofold:

1. To apply the method of joint generalized least squares to the ship cost problem.

2. Develop computer routines to carry out the calculations associated with the joint generalized least squares estimation technique.

III. METHODOLOGY OF JOINT GENERALIZED LEAST SQUARES ESTIMATION

Joint generalized least squares is a method for taking into consideration information that may be available when one is interested in the combination of several linear relations which are related either because their coefficients are partly the same or because their disturbances are correlated. In the latter case one can say that the variables that are neglected in the various equations are partly the same or at least correlated. The statistic used to measure the correlation between relations or subsystem errors is

$$\hat{\rho}_{12} = \frac{\sum_i (Y_{i1} - \hat{Y}_{i1})(Y_{i2} - \hat{Y}_{i2})}{\sqrt{\sum_i (Y_{i1} - \hat{Y}_{i1})^2 \sum (Y_{i2} - \hat{Y}_{i2})^2}}$$

as was defined earlier in the RAND study on uncertainty in aggregate predictions.

A. MODEL AND ASSUMPTIONS

A complete development of the Joint Generalized Least Squares Method is contained in Theil [Ref. 7]. In using the joint generalized least squares method it is desired to formulate the estimates of the parameter vectors of several subsystem equations simultaneously. Suppose there are L equations of the form

$$y_j = X_j\beta_j + \varepsilon_j \qquad j = 1,\ldots,L$$

where y_j is a column vector of n observations

X_j is an n x K_j matrix of values taken by the K_j explanatory variables of the jth subsystem relation

β_j is the corresponding parameter vector to be estimated

ε_j is a disturbance vector

Suppose also that each of these L subsystem equations meets the assumptions of the standard linear regression model. It is desired to combine the L equations in the following form:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} X_1 & & & 0 \\ & X_2 & & \\ & & \ddots & \\ 0 & & & X_L \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_L \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_L \end{bmatrix}$$

For notational ease let

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} \quad X = \begin{bmatrix} X_1 & & & 0 \\ & X_2 & & \\ & & \ddots & \\ 0 & & & X_L \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_L \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_L \end{bmatrix}$$

Then the joint generalized least squares formulation becomes:

$$Y = X\beta + \varepsilon$$

1. Assumptions for Joint Generalized Least Squares

a. The standard linear model holds for all of the L subsystem relations under consideration.

b. The disturbances of each subsystem equation are homescedastic and uncorrelated,

$$E(\varepsilon_j \varepsilon_j') = \sigma_{jj} I \qquad j = 1,\ldots,L$$

where σ_{jj} is the variance of the disturbance vector of the j^{th} equation.

c. Disturbances of different observations but of the same linear relation are assumed to be zero,

$$E(\varepsilon_{\alpha j} \varepsilon_{nj}) = 0 \qquad \alpha \neq n \quad j = 1,\ldots,L$$

d. With respect to the covariance matrix of the disturbances of two different equations:

$$E(\varepsilon_j \varepsilon_\ell') = \begin{bmatrix} E(\varepsilon_{1j}\varepsilon_{1\ell}) & E(\varepsilon_{1j}\varepsilon_{2\ell}) & \ldots & E(\varepsilon_{1j}\varepsilon_{n\ell}) \\ E(\varepsilon_{2j}\varepsilon_{1\ell}) & E(\varepsilon_{2j}\varepsilon_{2\ell}) & \ldots & E(\varepsilon_{2j}\varepsilon_{n\ell}) \\ \vdots & \vdots & & \vdots \\ E(\varepsilon_{nj}\varepsilon_{1\ell}) & E(\varepsilon_{nj}\varepsilon_{2\ell}) & \ldots & E(\varepsilon_{nj}\varepsilon_{n\ell}) \end{bmatrix}$$

The diagonal elements are contemporaneous covariances of the form $(\varepsilon_{\alpha j}\varepsilon_{\alpha \ell})$ and are assumed to be constant in the sense of being independent of α. These will be denoted by $\sigma_{j\ell}$. The off diagonal elements correspond to different observations and are assumed to be zero:

$$E(\varepsilon_{\alpha j}\varepsilon_{n\ell}) = 0 \qquad \begin{matrix} \alpha \neq n \\ j \neq \ell \end{matrix} \quad j, \ell = 1,\ldots,L$$

Note we obtain

$$E(\varepsilon_j \varepsilon_\ell') = \sigma_{j\ell} I \qquad j,\ell = 1,\ldots,L$$

which includes $E(\varepsilon_j \varepsilon_j) = \sigma_{jj} I$ as a special case.

The disturbances of the L linear relations for the same observation are random drawings from a multivariate population with zero mean and constant covariance matrix. Thus the covariance of the complete vector ε is:[1]

$$\text{VAR}(\varepsilon) = \begin{bmatrix} \sigma_{11}I & \sigma_{12}I & \cdots & \sigma_{1L}I \\ \sigma_{21}I & \sigma_{22}I & \cdots & \sigma_{2L}I \\ \vdots & \vdots & & \vdots \\ \sigma_{L1}I & \sigma_{L2}I & \cdots & \sigma_{LL}I \end{bmatrix} = \Sigma \otimes I$$

where $\Sigma = [\sigma_{j\ell}]$

The L x L matrix Σ is the covariance matrix of $[\varepsilon_{\alpha 1} \cdots \varepsilon_{\alpha L}]$ for any α.

The complete β vector may now be estimated by generalized least squares. The Aitken estimator for β is,

$$\hat{\beta} = [X'(\Sigma^{-1} \otimes I)X]^{-1} X'(\Sigma^{-1} \otimes I)Y$$

---

[1]The Kronecker product of the m x n matrix A and the p x q matrix B is defined as the mp x nq matrix

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & & \vdots \\ a_{n1}B & \cdots & a_{mn}B \end{bmatrix}$$

For further properties of the Kronecker product see [Ref. 7, p. 305].

for which the covariance matrix is,

$$VAR(\hat{\beta}) = [X'(\Sigma^{-1} \otimes I)X]^{-1}$$

Since Σ is usually unknown, it is estimated by the matrix of mean squares and products of the least squares residuals.

$$S = \frac{1}{n} \begin{bmatrix} e_1' \\ \vdots \\ e_L' \end{bmatrix} [e_1 \cdots e_L]$$

where e_j is the least squares residual vector of the j^{th} equation and n is the number of observations.

Replacing Σ with S we obtain the joint generalized least squares estimate of the parameter vector β

$$b_j = [X'(S^{-1} \otimes I)X]^{-1}X'(S^{-1} \otimes I)Y$$

with the estimator for the covariance matrix

$$VAR(b_j) = [X'(S^{-1} \otimes I)X]^{-1}$$

Thus we have the essential information to carry out the joint generalized least squares estimation technique.

B. TOTAL COST VARIANCE

Consider the estimator for Σ:

$$S = \frac{1}{n} \begin{bmatrix} e_1' \\ \vdots \\ e_L' \end{bmatrix} [e_1 \cdots e_L]$$

An estimate for total cost variance may be obtained by taking the summation of all the elements of S

$$\text{Var}(C) = i' S i$$ where i is a column vector of L ones

This estimate will be used in the analysis of the ship cost problem.

C. PREDICTION INTERVALS

1. Prediction Intervals Utilizing Joint Generalized Least Squares

A method for determining a prediction interval utilizing joint generalized least squares may be developed. It must be assumed that the disturbances of the L linear relations for the same observation are random drawings from a multivariate normal distribution with zero mean vector and a constant covariance matrix. Let $C = \sum_{j=1}^{L} Y_j$ where C is total cost; then

$$C \sim N\left(\sum_j X_{*j}\beta_j,\ i'\Sigma i\right)$$

where X_* is a row of explanatory variables $[X_{*1},\ldots,X_{*L}]$ for the L subsystem relations

Now if $\hat{C} = \sum_{j=1}^{L} \hat{Y}_j$ where $\hat{C}$ and $\hat{Y}_j$ are predicted costs, then

$$\hat{C} \sim N\left(\sum_j X_{*j}\beta_j,\ \text{VAR}\left(\sum X_{*j}\beta_j\right)\right)$$

Therefore

$$C - \hat{C} \sim N\left(0,\ i'\Sigma i + \text{VAR}\left(\sum X_{*j}\beta_j\right)\right)$$

If the estimates utilizing the sample observations are $i'Si$ for $i'\Sigma i$ and b_j, the joint generalized least squares estimate vector, for β, a prediction interval may be derived as

$$C \pm \eta_{(a/2)} \sqrt{i'Si + X_* \, VAR(b_j) X_*'}$$

2. Prediction Intervals for Correlated Subsystems Utilizing Least Squares Estimation Techniques

It is possible to provide prediction intervals by single equation least squares estimation technique under the circumstances of correlated residuals and different explanatory variables for each subsystem. When substantial correlations exist between subsystems in the model the below described methodology can be expected to give more accurate statements about uncertainty in aggregate cost predictions than would the normal practice of disregarding the correlations and without utilizing simultaneous estimates.

An estimated covariance matrix of the least squares estimates of the coefficients is obtained as will now be described. Consider the L subsystem equations of the form

$$y_j = X_j \beta_j + \varepsilon_j \qquad j = 1, \ldots, L$$

Postmultiply the sampling error of the least squares estimator, $\hat{\beta}_i$, of β_i by the transpose of the sampling error of the estimator, $\hat{\beta}_j$, of β_j. Then the covariance matrix for selections i and j may be described as

$$E[(\hat{\beta}_i - \beta_i)(\hat{\beta}_j - \beta_j)] = \sigma_{ij}(X_i'X_i)^{-1}X_i'X_j(X_j'X_j)^{-1}$$

where σ_{12} is the contemporaneous covariance,

X_i, X_j are observation matrices

$(\hat{\beta}_i - \beta_i)$ are sampling errors.

This result will hold if all of the assumptions concerning disturbances that were made for the joint generalized least squares method are accepted. These assumptions are much less restrictive than those of the least squares (independence) method.

Utilizing this concept the covariance matrices for all combinations of subsystem selections may be determined. The estimates for $\hat{\sigma}_{ij}$ for all subsystems are contained in S, the matrix of mean squares and products of the residuals. Letting $E_{ij} = E[(\hat{\beta}_i - \beta_i)(\hat{\beta}_j - \beta_j)]$ a covariance matrix, denoted VAR(β), similar to the covariance matrix of the joint estimates may be constructed in the form shown below

$$\begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1L} \\ E_{21} & E_{22} & \cdots & E_{2L} \\ \vdots & \vdots & E_{ij} & \vdots \\ E_{L1} & E_{L2} & \cdots & E_{LL} \end{bmatrix}$$

If it is assumed that the disturbances of the L linear relations for the same observation are random drawings from a multivariate normal distribution with zero mean vector and a constant covariance matrix, a method for determining a prediction interval may be developed paralleling the prediction interval method utilizing joint generalized least squares estimates. The resulting prediction interval becomes

$$\hat{C} \pm \eta_{(\alpha/2)} \sqrt{i'Si + X_* VAR(\hat{\beta}) X_*'}$$

where $\hat{C}$ is estimate of total cost

X_* is a row vector of explanatory variables $[X_{*1}, \ldots X_{*L}]$ for the L subsystem relations

$VAR(\hat{\beta})$ is the covariance matrix of the least squares estimates.

IV. DATA BASE

There are three subsets of the data base that require discussion; contractor bid cost data, end cost data, and the independent variables. Contractor bid data was used for predicting subsystem costs. This was a compromise, the reason for which was that bid cost data was the most meaningful data available for the period under study (1954-1966). Cost accounting systems differed greatly among the various contractors and NAVSHIPS, making it impossible to obtain data on a uniform level of aggregation from any other sources. Bid prices themselves are subject to fluctuations due to factors in the shipbuilding industry such as overhead distribution, workload, level of expertise at a particular shipyard. The source of the bid data was the file of NAVSHIPS Form 4282.2, UNIT PRICE ANALYSIS-BASIC CONSTRUCTION, which lists contractor estimates for the nine different construction cost groups, subdivided into three categories; direct labor, direct material and overhead costs. The source of end cost data was the NAVSHIPS records of Shipbuilding and Conversion, Navy (SCN) fund. There are forty-one physical and performance characteristics for each ship which are contained in the RMC data base. These characteristics are essentially design parameters such as maximum speed, maximum draft, hull, propulsion weights, etc. They are utilized as candidates for independent variables in the models developed. Reference 4 and reference 6 give more

detailed descriptions of the entire data base. Appendix A gives a description of Ship's Characteristics used as explanatory variables. Appendix B gives a description of basic contract cost categories.

A. DATA ADJUSTMENTS

1. Bid Cost Data

Contractor raw bid data was adjusted in four specific ways to remove cost variations due to factors other than ship's characteristics as follows:

a. Learning effect - when the cost of ship construction decreases progressively with each ship in a procurement lot.

b. Temporal effect - which takes into consideration variations of prices, productivity and wage overtime. 1965 indices were used.

c. Installation of government furnished equipment.

d. Cost of plans from external sources.

The order of adjustment was as follows:

(1) Application of learning curves.

(2) Adjustments using 1965 indices.

(3) Addition of the cost of GFE and plans.

Details of these adjustments are found in Ref. 4 and Ref. 6.

2. End Cost Data

End cost data was adjusted in much the same way that contractor bid data was adjusted. Three specific adjustments were made to the raw end cost data.

a. Treatment for learning effect.

b. Adjustment for temporal effect using 1965 indices.

c. An adjustment for nuclear technology with propulsion costs.

Details of these adjustments are found in Ref. 4 and Ref. 6. The data base is published separately as APPENDIX H of Ref. 4 and is a CONFIDENTIAL document.

B. DATA BASE STRATIFICATION AND COST GROUP MODELS

Hernon and McCumber noted that some DLG type ships had significantly higher costs in the areas of hull, outfitting, construction services, weapons end cost, and electronics end cost. In addition the DLG ship was considered to have a different operational mission than the smaller, less expensive destroyer type ships. Thus, two basic data base stratification levels were examined:

1. General Data Base: DD/DDG/DE/DEG/DLG (36 Ships)
2. Escort Data Base : DD/DDG/DE/DEG (27 Ships)

Using both of the data bases, CERs were developed using two different methods of cost disaggregation schemes which are examined in this paper. The coding system as developed by Hernon and McCumber is contained in Table I and will be used for easy reference.

TABLE I

DATA BASE STRATIFICATION LEVELS AND COST GROUP MODELS

	CER CODE	
	General Level	Escort Level
MODELS B/C - 9 COST GROUP CERS		
Hull Cost	B-1	C-1
Propulsion Cost	B-2	C-2
Electrical Cost	B-3	C-3
Communication & Control + Electronics End Costs	B-4	C-4
Auxiliary Cost	B-5	C-5
Outfitting Cost	B-6	C-6
Armament + Weapons End Costs	B-7	C-7
Design & Engineering Cost	B-8	C-8
Construction Services + Miscellaneous End Costs	B-9	C-9
MODELS D/E - 4 COST GROUP CER		
Base Cost = Hull + Outfitting	D-1	E-1
Engineering Cost = Propulsion + Electrical + Auxiliary	D-2	E-2
Payload Cost = C & C + Armament + Electronics End Cost + Weapons End Cost	D-3	E-3
Construction Cost = D & E + Construction Services + Miscellaneous End Cost	D-4	E-4

V. ANALYSIS

The general scheme of analysis was to apply the joint generalized least squares method as has been developed to the ship cost problem. In doing so computer routines were developed to carry out the indicated calculations associated with joint generalized least squares. Computer routines were also developed to solve the problem utilizing the least squares method in order to compare the results of the two methods and more clearly determine the nature of gains by using the joint generalized least squares method.

A. GENERAL COMMENTS

1. Computer Programs

A computer program was developed in order to perform the joint generalized least squares analysis. Essentially, a data set is read in and then transformations to the data as programmed are conducted. The program then does the number of standard multiple linear regressions required (one for each subsystem CER). The residuals from the single regressions are saved and utilized to compute a matrix of correlations between selections. Next the program does the joints generalized least squares calculations; initially S is computed from the least squares residuals, next b_j, the parameter estimate vector is computed along with the covariance matrix $V(b_j)$. Finally, the joint generalized least squares prediction interval values are calculated.

Details of the program and a users manual are contained in APPENDIX D. The program was designed to be easy to use for analysis utilizing the joint generalized least squares method. Double precision was used to improve accuracy by reducing roundoff errors. The program will handle up to thirty-six observations and nine subsystem relations. Each subsystem relation may have up to four explanatory variables (including the constant term). It should be noted that larger dimensioned problems may be possible but computer capacity becomes a constraint to be concerned with.

A computer program was also developed to perform the calculations required to compute a least squares prediction interval assuming independence between subsystem CERs. Essentially a data set is read in and then transformations to the data as programed are conducted. The program then does the number of standard multiple linear regressions required (one for each subsystem CER). Then the information required from each individual regression is used in the calculation of the prediction interval. This computer program is provided in Appendix D.

Finally, a computer program was developed to perform the calculations required to compute a least squares prediction interval assuming correlated disturbances between subsystem CERs. Essentially, a data set is read in and transformations as programed performed. Next the covariance matrix of the least squares estimates as described in

Section III.C.2 is constructed. Finally the calculations are carried out to compute least squares correlated prediction interval values. This computer program is provided in Appendix D.

2. Criteria

The principle criteria used for evaluating the joint generalized least squares method were the reduction in the covariance matrix of the estimate, the prediction intervals computed and the mean square residual values obtained from each observation. The first, a comparison of the diagonal elements of the estimated covariance matrix of the joint estimates and the estimated covariance matrix of the least squares estimates gives an impression of the gain obtained by applying the joint generalized least squares method rather than the least squares method. The second, the prediction intervals computed, gives the effect of the assumption of independent subsystem CER disturbances as compared to the assumption of correlated subsystem CER disturbances, and also a feel for the effect of using the joint generalized least squares estimation technique. In the third, mean squared residual values (MSR) are obtained from each observation, as the difference between observed and total cost. When these total cost residuals are squared, summed for all observations (ships), and corrected for the number of

observations, a statistic is obtained for comparison of a set of models.[2]

3. Heteroscedasticity

Hernon and McCumber used a log linear model for their propulsion and engineering CERs in their nine CER group model and four CER group model respectively. This was done in order to correct for heteroscedasticity encountered with the linear model. The joint generalized least squares method will not allow calculation of prediction intervals when individual CERs are of mixed linear and log linear form, therefore, an alternate method was used to correct for heteroscedasticity in these relationships.

Consider the relation $y_\alpha = \beta_0 + \beta_1 X_{\alpha 1} + \varepsilon_\alpha$ for $\alpha = 1,\ldots,n$ where ε_α is a random disturbance with zero expectation. If it is assumed that the disturbance variance is proportional to the square of the independent variable then $\sigma_\alpha^2 = kX_{\alpha 1}^2$ for $\alpha = 1,\ldots,n$ and for some positive constant

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[2]Reference is made to mean squared residual (MSR) values in two contexts in this paper. MSR values refer to the sum of the squared total cost residuals for each observation divided by the number of observations. MSR values adjusted for degrees of freedom refer to the sum of the squared total cost residuals for each observation divided by the number of observations minus the number of parameters estimated minus the number of CERs utilized in the model. The first is a statistic for comparison between models, where as the second is an upper bound estimate on total cost variance as described by Hernon and McCumber [Ref. 4].

k. Dividing the values of the α^{th} observation by $X_{\alpha 1}$ is thus equivalent to the following reformulation:

$$\frac{y_\alpha}{X_{\alpha 1}} = \frac{\beta_0}{X_{\alpha 1}} + \beta_1 + \frac{\varepsilon_\alpha}{X_{\alpha 1}}$$

The disturbances $\varepsilon_1/X_{11} \ldots \varepsilon_n/X_{n1}$ have a scalar covariance matrix (kI) under the formulation so that the assumptions of the standard linear model are now satisfied. Scatter plots of residuals versus independent explanatory variables were used to determine the variable causing the heteroscedasticity in each relation and the above described reformulation was used to correct for it. A full description of the method is found in Theil [Ref. 7].

4. Number of Observations

Hernon and McCumber conducted an analysis of residuals to verify the normality assumption of the least squares regression technique and to identify any significant outliers. This resulted in the deletion of outliers in the development of several subsystem CERs. This caused the observations of a particular ship to be used to calculate some subsystem CERs, while not being used in others. Joint generalized least squares requires the same number of observations for all subsystem relations. Therefore, all observations were used in the joint generalized least squares analysis. In Appendix B of Ref. 4 Hernon and McCumber report the CERs developed including outliers in the data base. For purposes of comparison with joint generalized least squares results

these CERs were used instead of the CERs developed by the deletion of outliers. Thus the general data base has thirty-six observations (ships) and the escort data base has twenty-seven observations.

B. COST GROUP CER MODELS ANALYSIS

The nine cost group CER model consists of CERs as indicated and coded in Table 1. The regression strategy used in the development of these CERs along with a discussion of the results is found in Ref. 4. It was necessary to use a scale factor of 1/100 for the variables ENGPAY, PWRLD, ELEWGT, C&CWGT, PRAXWT, LSW, ARMWGT and PROLSW (the weight variables used) to enable inversion of the matrices required in calculating the joint generalized least squares estimates. These factors are reflected in the computer output in Appendix C for the mine group CER models, but have been removed in the results reported in this section.

The four cost group CER model consists also of CERs as indicated and coded in Table 1. No scale factor was required. In deriving the condensed model an attempt was made to make the subsystem categories as independent as possible, first with respect to the nature of ship design, and secondly with respect to the differences reported earlier in shipyard and contractor accounting practices. Details of the regression strategy and development of the subsystem CERs are contained in Hernon and McCumber,[Ref. 4].

1. Results

The following tables present four different sets of CERs - one set for each of the two models employing the general data base, and one set for each of the same two models employing the reduced escort data base. Following each set of CERs are tables containing correlations between selections for that particular model and selective comparisons of covariance matrices of the least squares and the joint generalized least squares estimates. The complete covariance matrices are contained in Appendix C, computer output.

TABLE II: MODEL B - 9 SUBSYSTEM CERs - GENERAL DATA BASE			
CODE	METHOD	COST SUB-CATEGORY	CER
B-1	LS	Hull	Cost = .0406 + .00139 [ENGPAY]
	JGLS		Cost = .0831 + .00136 [ENGPAY]
B-2	LS	Propulsion	Cost = -6.0916 + .5867 [PWRLD] + .000729 [RANGE]
	JGLS		Cost = -5.925 + .5870 [PWRLD] + .000697 [RANGE]
B-3	LS	Electrical	Cost = .1486 + .00386 [ELEWGT] + .2616 [NO-GEN]
	JGLS		Cost = .0310 + .00352 [ELEWGT] + .3099 [NO-GEN]
B-4	LS	C&C + Electronics	Cost = -.9037 + .032 [C&C WGT] + .348 [PROTO]
	JGLS	End Cost	Cost = -.6208 + .029 [C&C WGT] + .627 [PROTO]
B-5	LS	Auxiliary	Cost = .0263 + .00224 [PRAXWGT]
	JGLS		Cost = -.0055 + .00227 [PRAXWGT]
B-6	LS	Outfitting	Cost = .436 + .000386 [LSW]
	JGLS		Cost = .549 + .000350 [LSW]
B-7	LS	Armament + Weapons	Cost = -2.608 + .0376 [ARMWGT] + 9.178 [MS-END]
	JGLS	End Cost	Cost = -2.576 + .0884 [ARMWGT] + 3.859 [MS-END]
B-8	LS	Design and	Cost = -1.0125 + .00654 [ARMWGT] + .00151 [PROLSW]
	JGLS	Engineering	Cost = -.9260 + .00638 [ARMWGT] + .00144 [PROLSW]
B-9	LS	Construction Services + Miscellaneous End Cost	Cost = 3.138 + .0394 [C&CWGT] - 59.846 [AR/LSW] + .00059 [PROLSW]
	JGLS		Cost = 2.520 + .0366 [C&CWGT] - 42.153 [AR/LSW] + .00052 [PROLSW]

TABLE III: MATRIX OF CORRELATIONS BETWEEN SELECTIONS
MODEL B - 9 CERs - GENERAL DATA BASE

Hull	Prop	Elect	C&C	Auxiliary	Out	Arm	D&E	Const	
1.0	.084	-.227	-.141	.359	-.021	-.094	.106	.102	Hull
	1.0	-.109	-.105	-.238	.424	.263	.160	.252	Propul-sion
		1.0	.194	.234	-.050	-.158	-.023	.180	Elec-trical
			1.0	-.071	-.405	.139	.115	.402	C&C + Elect.
				1.0	.221	-.152	.057	.101	Auxil-iary
					1.0	.322	.021	.059	Outfit-ting
						1.0	.252	.362	Arm + Weapons
							1.0	.234	D&E
								1.0	Const. Serv.

TABLE IV

BLOCK DIAGONAL COVARIANCE MATRIX COMPARISONS
MODEL B - 9 CERs - GENERAL DATA BASE

JGLS			LS		
HULL					
.7293D-01	-.3523D-02		.7392D-01	-.3578D-02	
	.1960D-03			.1991D-03	
PROPULSION					
.1244D 02	-.1994D 01	.1330D-03	.1499D 02	-.2270D 01	.1294D-03
	.5012D 00	-.5009D-04		.5734D 00	-.5755D-04
		.6428D-08			.7885D-08
ELECTRICAL					
.3025D-01	-.1964D-02	-.7572D-02	.3377D-01	-.1597D-03	-.9330D-02
	.7613D-02	-.2417D-02		.9372D-02	-.3647D-02
		.3206D-02			.4214D-02
COMMUNICATION AND CONTROL					
.2177D 01	-.1032D 01	-.4362D 00	.2381D 01	-.1112D 01	-.7143D 00
	.6008D 00	.9698D-01		.6405D 00	.1518D 00
		.1095D 01			.1839D 01
AUXILIARY					
.6227D-01	-.5399D-02		.6439D-01	-.5612D-02	
	.5425D-03			.5639D-03	
OUTFITTING					
.1808D-01	-.4824D-03		.1978D-01	-.5352D-03	
	.1510D-04			.1675D-04	

TABLE V: MODEL C - 9 SUBSYSTEM CERs - ESCORT DATA BASE

CODE	METHOD	COST SUB-CATEGORY	CER
C-1	LS	Hull	Cost = .5524 + .00098 [ENGPAY]
	JGLS		Cost = .46877 + .00103 [ENGPAY]
C-2	LS	Propulsion	Cost = -.49215 + .38066 [PWRLD]
	JGLS		Cost = -.54098 + .38494 [PWRLD]
C-3	LS	Electrical	Cost = -.0151 + .00657 [ELEWGT] + .23057 [NO-GEN]
	JGLS		Cost = .07492 + .00524 [ELEWGT] + .24605 [NO-GEN]
C-4	LS	C&C + Electronics	Cost = -.4459 + .03429 [C&CWGT] - 1.2848 [MS-END]
	JGLS	End Cost	Cost = -.9609 + .03851 [C&CWGT] - 1.4058 [MS-END]
C-5	LS	Auxiliary	Cost = -.0007 + .00229 [PRAXWT]
	JGLS		Cost = -.20962 + .00254 [PRAXWT]
C-6	LS	Outfitting	Cost = .5713 + .00332 [OUTWGT]
	JGLS		Cost = .6213 + .00311 [OUTWGT]
C-7	LS	Armament + Weapons	Cost = -1.00 + 7.9916 [MS-END] + .02814 [ARMWGT]
	JGLS	End Cost	Cost = -1.392 + 7.7317 [MS-END] + .03110 [ARMWGT]
C-8	LS	Design and	Cost = -.3065 + .00407 [ARMWGT] + .00094 [PROLSW]
	JGLS	Engineering	Cost = -.1182 + .00306 [ARMWGT] + .00092 [PROLSW]
C-9	LS	Construction	Cost + 3.7417 + .0394 [C&CWGT] - 68.6435 [AR/LSW]
	JGLS	Services +	+ .00039 [PROLSW]
		Miscellaneous	Cost = 3.2207 + .0374 [C&CWGT] - 57.5419 [AR/LSW]
		End Cost	+ .00051 [PROLSW]

TABLE VI: MATRIX OF CORRELATIONS BETWEEN SELECTIONS
MODEL C - 9 CERs - ESCORT DATA BASE

Hull	Prop	Elect	C&C	Auxiliary	Out	Arm	D&E	Const	
1.0	-.035	.029	.001	.382	-.418	-.060	.0884	-.104	Hull
	1.0	.004	.009	-.449	-.291	-.013	.148	.000	Propulsion
		1.0	-.040	.283	.123	-.230	.034	.187	Electrical
			1.0	-.010	-.50	.062	.063	.577	C&C + Elect.
				1.0	-.206	-.058	-.019	.049	Auxiliary
					1.0	.019	.006	-.193	Outfitting
						1.0	.049	.306	Arm + Weapons
							1.0	.196	D&E
								1.0	Construction Services

TABLE VII

BLOCK DIAGONAL COVARIANCE MATRIX COMPARISONS
MODEL C - 9 CERs - ESCORT BASE

JGLS			LS		
HULL					
.4421D-01	-.2579D-02		.4831D-01	-.2850D-02	
	.17130D-03			.1894D-03	
PROPULSION					
.5893D 01	-.5540D 00		.7706D 01	-.7609D 00	
	.6322D-01			.8684D-01	
ELECTRICAL					
.2340D-01	-.1152D-01	-.3137D-02	.2498D-01	-.1232D-01	-.3373D-02
	.2160D-01	-.3461D-02		.2770D-01	-.5219D-02
		.2137D-02			.2792D-02
COMMUNICATION AND CONTROL					
.3781D 01	-.2604D 01	.5018D 00	.4243D 01	-.3150D 01	.1083D 01
	.2230D 01	-.8792D 00		.3007D 01	-.1932D 01
		.1436D 01			.3200D 01
AUXILIARY					
.8039D-01	-.8081D-02		.9440D-01	-.9740D-02	
	.9571D-03			.1154D-02	
OUTFITTING					
.2786D-01	-.1039D-03		.3586D-01	-.1363D-03	
	.4208D-06			.5520D-06	

TABLE VIII: MODEL D - 4 SUBSYSTEM CERs - GENERAL DATA BASE

CODE	METHOD	COST SUB-CATEGORY	CER
D-1	LS	Base	Cost = .4708 + .00115 [LSW]
	JGLS		Cost = .5979 + .00111 [LSW]
D-2	LS	Engineering	Cost = -.6515 + .00522 [PWRLD] + .2864 [ENGWGT]
	JGLS		Cost = -.5748 + .00529 [PWRLD] + .2745 [ENGWGT]
D-3	LS	Payload	Cost = -6.7857 + .00585 [LSW] + 6.6641 [MS-END]
	JGLS		Cost = -6.3078 + .00558 [LSW] + 7.1679 [MS-END]
D-4	LS	Construction	Cost = -1.0140 + .00555 [HULWGT] + .00201 [PROLSW]
	JGLS		Cost = -.6801 + .00540 [HULWGT] + .00183 [PROLSW]

MATRIX OF CORRELATIONS BETWEEN SELECTIONS
MODEL D - 4 CERs - GENERAL DATA BASE

Base	Engin.	Pay	Const	
1.0	.441	-.106	-.055	Base
	1.0	.085	.279	Engineering
		1.0	.555	Payload
			1.0	Construction

TABLE IX

BLOCK DIAGONAL COVARIANCE MATRIX COMPARISONS
MODEL D - 4 CERs - GENERAL DATA BASE

JGLS			LS		
BASE D-1					
.1197D 00	-.3214D-04		.1251D 00	-.3384D-04	
	.1006D-07			.1059D-07	
ENGINEERING D-2					
.1539D-06	-.1136D-04	.9565D-05	.2336D-06	-.1905D-04	.3201D-04
	.1745D-02	-.9690D-02		.3165D-02	-.1898D-01
		.1022D 00			.1882D 00
PAYLOAD D-3					
.5081D 01	-.1726D-02	.1422D 01	.5498D 01	-.1991D-02	.2016D 01
	.7672D-06	-.1004D-02		.9495D-06	-.1444D-02
		.2470D 01			.3598D 01
CONSTRUCTION D-4					
.7468D 00	-.4696D-03	.7886D-06	.7844D 00	-.4966D-03	-.1791D-05
	.3680D-06	-.2255D-07		.3946D-06	.3358D-07
		.3938D-07			.6266D-07

TABLE X: MODEL E - 4 SUBSYSTEM CERs - ESCORT DATA BASE

CODE	METHOD	COST SUB-CATEGORY	CER
E-1	LS	Base	Cost = 1.2072 + .00083 [LSW]
	JGLS		Cost = 1.0728 + .00088 [LSW]
E-2	LS	Engineering	Cost = -.7559 + .66217 [PWRLD]
	JGLS		Cost = -.9403 + .67833 [PWRLD]
E-3	LS	Payload	Cost = -4.1883 + .00468 [LSW] + 7.1489 [MS-END]
	JGLS		Cost = -4.7791 + .00478 [LSW] + 7.8049 [MS-END]
E-4	LS	Construction	Cost = -2.6274 + .00736 [HULWGT] + .00156 [PROLSW]
	JGLS		Cost = -2.0501 + .00679 [HULWGT] + .00156 [PROLSW]

MATRIX OF CORRELATIONS BETWEEN SELECTIONS
MODEL E - 4 CERs - ESCORT DATA BASE

Base	Engin.	Pay	Const	
1.0	-.255	-.280	-.318	Base
	1.0	-.043	.070	Engineering
		1.0	.490	Payload
			1.0	Construction

TABLE XI

BLOCK DIAGONAL COVARIANCE MATRIX COMPARISONS
MODEL E - 4 CERs - ESCORT DATA BASE

JGLS			LS		
BASE E-1					
.1056D 00	-.3761D-04		.1108D 00	-.3962D-04	
	.1456D-07			.1533D-07	
ENGINEERING E-2					
.1479D-02	-.1452D-01		.1533D-02	-.1513D-01	
	.1657D 00			.1727D 00	
PAYLOAD E-3					
.1052D 02	-.4401D-02	.3108D 01	.1179D 02	-.5108D-02	.4255D 01
	.2065D-05	-.1943D-02		.2472D-05	-.2658D-02
		.3970D 01			.5429D 01
CONSTRUCTION E-4					
.2226D 01	-.1993D-02	-.5692D-04	.2374D 01	-.2120D-02	-.8667D-04
	.1932D-05	.3906D-08		.2050D-05	.1330D-07
		.8972D-07			.1238D-06

The following table indicates correlations of greater than ± .4 for the four models:

TABLE XII

MODEL	CORRELATION	SUBSYSTEMS
B	.424	Outfitting and Propulsion
	- .405	Outfitting and C&C + Electrical
	.402	Construction and C&C + Electrical
C	- .449	Auxiliary and Propulsion
	- .418	Outfitting and Hull
	- .50	Outfitting and C&C + Electrical
	.577	Construction and C&C + Electrical
D	.441	Engineering and Base
	.555	Construction and Payload
E	.490	Construction and Payload

These correlation estimates do indicate significant degrees of correlation between subsystem CERs and would tend to provide evidence against a hypothesis of independence between subsystem CERs.

The block diagonal covariance matrix comparisons show a reduction in the variance (diagonal) elements in all cases and a reduction in the covariance (off-diagonal) elements in the large majority of cases by the joint generalized least squares estimates as compared with the least squares estimates. This may be attributed to the fact that the joint

generalized least squares method takes into consideration the correlated subsystem disturbances in computing the joint estimates whereas the least squares method computes the individual subsystem estimates separately.

2. Total Cost Variance

Total cost variance for each model was computed utilizing three methods. The first method was to take the summation of the individual subsystem CER standard errors assuming independent subsystems. The second method was to take the summation of the entries in the S matrix of mean squares and products of all sets of subsystem CER residuals, the estimate for Σ. The third estimate was obtained by summing the residual values produced by each CER for a given ship which gives the difference between observed and predicted cost for that ship as an aggregate of the individual CER values. The total cost residuals are then squared and summed for all observations (ships) and divided by the degrees of freedom, defined as the number of observations minus the number of parameters estimated minus the number of subsystem CERs, to obtain the mean squared residual value (MSR). This method was used by Hernon and McCumber [Ref. 4] to form an upper bound on total cost variance. In essence, the summation method assuming independence provides a lower bound on total cost variance. The summation of the S matrix elements (S matrix summation), assuming correlated subsystems provides a midrange, more realistic estimate, between the upper bound (MSR method) and lower bound (LS Summation method). The

values for these three methods are contained in the following table:

TABLE XIII

MODEL	METHOD (LS SUMMATION)	(S MATRIX SUMMATION)	(MSR)
B	19.64	123.9	163.0
C	16.12	58.38	526.0
D	7.86	36.54	58.3
E	7.04	27.47	43.6

3. Prediction Interval and Predicted Costs

Three prediction intervals were obtained for each model in the manner described in Section III.C, e.g., least squares with independence, least squares with correlations and joint generalized least squares with correlations. The estimated total ship cost was computed with the least squares estimates in the first two cases and the joint estimates in the third. The results are summarized in the following tables. There is a table for each model which contains the prediction interval length for each observation by each method. Also contained in each table are the absolute differences between actual and estimated cost obtained by using the least squares estimates and the joint estimates. The actual prediction intervals are contained in Appendix C. The signs of the value of the differences between actual

TABLE XIV

MODEL B: PREDICTION INTERVAL SUMMARY

OBS	LENGTH LS(IND)	LENGTH JGLS	LENGTH LS(CORR)	|Δ|LS ACTUAL-ESTIMATED	|Δ|JGLS ACTUAL-ESTIMATED
1	24.64	28.76	29.19	10.99	11.69
2	24.23	28.54	29.10	3.47	4.33
3	24.28	28.55	28.82	3.75	4.62
4	23.82	28.37	28.58	4.09	3.98
5	23.37	28.12	28.15	1.92	2.19
6	23.39	28.12	28.15	1.89	2.23
7	23.38	28.11	28.15	.65	.32
8	23.39	28.12	28.15	2.84	3.19
9	23.38	28.12	28.15	7.68	7.93
10	23.39	28.12	28.15	5.13	5.47
11	23.39	28.12	28.15	9.94	9.60
12	23.39	28.12	28.15	3.70	4.04
13	23.37	28.12	28.15	4.83	5.04
14	23.37	28.12	28.14	8.36	8.56
15	23.97	28.85	29.02	.46	1.23
16	23.78	28.74	28.84	2.80	2.01
17	23.78	28.74	28.84	3.37	2.58
18	23.78	28.74	28.84	3.78	2.99
19	24.07	29.12	29.33	2.15	2.37
20	23.89	28.99	29.08	5.33	5.18
21	23.84	28.84	29.04	2.31	2.26
22	23.70	28.51	28.72	1.49	1.16
23	23.34	28.27	28.34	3.43	3.60
24	23.33	28.27	28.34	24.15	24.30
25	23.36	28.29	28.36	1.08	.78
26	24.22	28.78	29.13	.78	1.53
27	23.79	28.48	28.68	4.81	4.22
28	23.56	28.34	28.40	18.37	18.39
29	23.56	28.34	28.40	3.0	2.99
30	24.56	29.44	29.59	18.28	19.39
31	24.12	28.99	29.15	6.45	7.29
32	23.85	28.69	28.81	5.37	4.76
33	23.85	28.69	28.81	1.77	1.16
34	24.48	29.16	29.60	10.82	12.26
35	23.19	28.83	28.93	.20	1.25
36	23.92	28.84	28.85	2.81	3.87

TABLE XV

MODEL C: PREDICTION INTERVAL SUMMARY

OBS	LENGTH LS(IND)	LENGTH JGLS	LENGTH LS(CORR)	|Δ|LS ACTUAL-ESTIMATED	|Δ|JGLS ACTUAL-ESTIMATED
1	22.02	23.38	23.90	6.96	8.0
2	21.73	23.17	23.56	2.02	2.79
3	21.76	23.17	23.57	2.19	2.93
4	20.71	23.02	23.35	10.58	10.05
5	20.92	22.88	22.98	1.89	1.62
6	20.92	22.82	22.95	1.83	1.59
7	20.91	22.81	22.95	4.32	4.10
8	20.92	22.82	22.95	.89	.64
9	20.92	22.87	22.98	3.9	4.17
10	20.92	22.82	22.95	1.41	1.65
11	20.92	22.82	22.95	13.67	13.42
12	20.92	22.82	22.95	.03	.22
13	20.91	22.86	22.97	1.07	1.31
14	20.91	22.88	22.97	4.62	4.86
15	21.30	23.46	23.58	1.43	1.18
16	21.29	23.45	23.56	.41	.79
17	21.29	23.45	23.56	.98	1.36
18	21.29	23.45	23.56	1.39	1.77
19	21.38	23.66	23.77	3.04	2.35
20	21.38	23.68	23.79	3.01	3.48
21	20.87	23.07	23.17	.24	.76
22	21.19	22.97	23.35	1.80	1.75
23	21.05	22.87	23.17	.60	.95
24	21.04	22.86	23.17	21.46	21.77
25	21.30	23.01	23.40	3.96	3.58
26	21.28	23.15	23.38	2.95	3.31
27	21.11	23.05	23.24	5.05	4.44

TABLE XVI

MODEL D: PREDICTION INTERVAL SUMMARY

OBS	LENGTH LS(IND)	LENGTH JGLS	LENGTH LS(CORR)	|Δ|LS ACTUAL-ESTIMATED	|Δ|JGLS ACTUAL-ESTIMATED
1	22.49	25.63	25.75	7.33	6.70
2	22.37	25.55	25.68	.44	.33
3	22.36	25.55	25.68	.51	.39
4	22.07	25.38	25.45	7.01	7.30
5	21.97	25.32	25.35	.69	.43
6	21.95	25.32	25.35	.39	.15
7	21.93	25.32	25.36	3.02	2.74
8	21.97	25.32	25.35	.54	.81
9	21.99	25.31	25.35	5.05	5.34
10	21.97	25.31	25.35	2.84	3.10
11	21.97	25.31	25.35	12.22	11.97
12	21.97	25.31	25.35	1.40	1.67
13	21.99	25.31	25.33	2.63	2.86
14	21.97	25.31	25.34	6.07	6.31
15	22.07	25.86	25.90	.84	1.05
16	22.04	25.85	25.88	1.90	1.46
17	22.04	25.85	25.88	2.47	2.03
18	22.04	25.85	25.88	2.89	2.44
19	22.71	25.82	25.86	2.65	2.88
20	21.99	25.80	25.82	4.33	3.86
21	21.88	25.58	25.92	.27	.24
22	22.04	25.53	25.65	4.81	4.27
23	21.93	25.46	25.55	.19	.14
24	21.93	25.46	25.54	20.54	20.57
25	21.95	25.46	25.55	4.0	3.96
26	22.00	25.43	25.58	1.49	1.54
27	21.88	25.42	25.50	4.86	5.29
28	22.10	25.41	25.44	19.61	19.67
29	22.10	25.41	25.44	1.76	1.69
30	23.56	26.27	26.52	18.78	19.54
31	23.82	26.23	26.39	9.08	8.98
32	23.26	26.16	26.34	5.74	5.88
33	23.26	26.16	26.34	2.14	2.28
34	24.39	27.04	27.40	4.0	5.59
35	24.09	25.87	27.11	5.25	4.68
36	24.09	25.87	27.11	2.59	2.01

TABLE XVII

MODEL E: PREDICTION INTERVAL SUMMARY

OBS	LENGTH LS(IND)	LENGTH JGLS	LENGTH LS(CORR)	|Δ|LS ACTUAL-ESTIMATED	|Δ|JGLS ACTUAL-ESTIMATED
1	21.39	21.77	22.03	4.70	4.59
2	21.15	21.58	21.79	.88	1.04
3	21.20	21.55	21.79	.48	.63
4	20.80	21.47	21.59	9.35	8.91
5	20.49	21.32	21.35	2.07	1.79
6	20.49	21.29	21.33	1.68	1.27
7	20.48	21.29	21.34	2.74	3.37
8	20.47	21.29	21.33	.74	.32
9	20.49	21.32	21.36	3.39	3.79
10	20.47	21.29	21.33	1.56	1.97
11	20.47	21.29	21.33	13.53	13.10
12	20.47	21.29	21.33	.12	.53
13	20.50	21.34	21.36	.90	1.30
14	20.50	21.34	21.37	4.54	4.93
15	20.88	22.05	22.11	2.56	2.29
16	20.88	22.04	22.11	.48	.20
17	20.88	22.04	22.11	.07	.37
18	20.88	22.04	22.11	.50	.78
19	20.59	21.94	22.00	2.52	2.16
20	20.59	21.92	21.99	3.85	4.24
21	20.13	21.39	21.44	.99	1.34
22	20.71	21.59	21.80	3.25	2.85
23	20.44	21.39	21.53	.14	.57
24	20.42	21.38	21.53	20.77	21.21
25	20.42	21.38	21.53	3.65	3.20
26	20.57	21.46	21.61	2.72	2.47
27	20.31	21.34	21.45	4.71	4.92

and estimated cost for both the least squares and the joint generalized least squares estimates are the same in all cases and have been dropped in the table for simplification.

The prediction interval length becomes longer progressing from least squares (independent) to joint generalized least squares and finally to least squares (correlated). Both least squares (correlated) and joint generalized least squares provide intervals which are significantly longer than those produced by the least squares (independent) method, while the joint generalized least squares method provides a small decrease in prediction interval length over the least squares (correlated) method. The latter two methods represent the prediction interval length more accurately assuming that there are correlations between subsystem CERs. Both of these methods require approximately the same degree of computational difficulty and thus it would appear that the joint generalized least squares technique offers the best alternative in accurately stating prediction intervals.

The mean square residual [MSR] value, not adjusted for degrees of freedom, was computed for the least squares estimates as compared to actual cost as was the same value for the joint estimates. It was intended to use these values as a way of comparing the two methods to see which provided the best estimates. But further examination led to the conclusion that there is no guarantee in using this technique that one method will be consistently better than the other.

In the joint generalized least squares case with heteroscedasticity the formulation of the problem is such that the length of the residual vector, after a transformation, is being minimized. Hence the solution we obtain is for the problem:

$$\text{Min } (Y - X\hat{\beta})(\Sigma^{-1} \otimes I)(Y - X\hat{\beta})$$

where $(\Sigma^{-1} \times I)$ is an unknown nonsingular matrix which is estimated by $(S^{-1} \times I)$

and for which $b_j = [X'(S^{-1} \otimes I)X]^{-1}X'(S^{-1} \otimes I)Y$ is the joint generalized least squares estimator as developed in Section III. This estimator, b_j, in fact does not minimize the length of the residual vector. Therefore the MSR method will not discriminate between the two methods of estimation in this case. For a complete discussion of this problem see Theil [Ref. 7].

The MSR results are contained in the following table:

TABLE XVIII

MODEL

	B	C	D	E
LS MSR	40.4	27.5	49.7	58.3
JGLS MSR	50.4	32.0	61.2	36.2

In models B, C and D the least squares estimates come out ahead whereas in model E the joint estimates come out ahead.

The total cost predictions by the least squares estimates and the joint estimates are very close when compared to each other. The Summary Tables present these values for comparison. On a worst case basis the differences between these two methods for each model, indicating the particular observation, are as follows:

MODEL/OBSERVATION

	B/34	C/1	D/36	E/7
SIZE OF DIFFERENCE $/MILLION	1.44	1.04	1.59	.63

In no case is this difference greater than three percent of total estimated cost.

Utilizing the additional information available due to the correlations between subsystems, joint generalized least squares estimates should provide a more accurate weighting of the variables in each CER. This is important in assessing the contribution to subsystem cost of a particular variable. For example, consider the CER B-6:

OUTFITTING COST = .436 + .000386 [LSW] (Least Squares)

OUTFITTING COST = .549 + .000350 [LSW] (Joint Generalized Least Squares)

The coefficient of light ship weight (LSW) reflects the change in outfitting cost per pound for a change in light ship weight. The joint generalized least squares coefficient

should provide a better reflection of the contribution of light ship weight to outfitting cost for this CER.

VI. CONCLUSIONS

It has been demonstrated that the joint generalized least squares estimation technique, by taking advantage of the additional information which is provided when correlations exist between the residuals of subsystem CERs in aggregate cost estimating, provides more accurate and meaningful statements about predictions, the total cost variance surrounding the predictions, and the prediction intervals. The method involves an increase in computational difficulty when compared to one assuming independence between subsystem CERs, but gives a more accurate statement concerning the estimates and their uncertainty. A computer program has been provided which is readily usable in conducting joint generalized least squares analysis and the additional time required would be minimal.

The predictions utilizing the joint estimates provide gains in a more accurate weighting of the variables in the subsystem CERs. This allows a better factor analysis of the contributions to subsystem cost and hence to total cost.

The estimate for total cost variances used in the joint generalized least squares method provide a more meaningful statement taking into consideration the correlated disturbances. The least squares estimate assuming independence tends to substantially understate the total cost variance in the situation analyzed.

The prediction intervals obtained utilizing the joint generalized least squares method represented a slight gain over those obtained by the least squares method, assuming correlated disturbances. Both these methods provided more accurate prediction intervals under the assumptions made; however, with equal computational difficulty the joint generalized least squares method appears the superior of the two.

It should be emphasized that the joint generalized least squares method involves less restrictive assumptions than the least squares method. The assumption of correlations between subsystem CERs is a more reasonable statement of the true relationship as was shown by the tables of correlations between subsystem CERs. The least squares method does in fact tend to understate the uncertainty surrounding the cost estimate. Joint generalized least squares by utilizing this additional information gives more meaningful results in the derivation of the estimate and the statement of uncertainty surrounding the estimate.

The overall gains achieved by the joint generalized least squares method overshadow the small additional cost. This application to destroyer type ship models may be extended to any other situation where one is interested in aggregate predictions of the same nature.

APPENDIX A

DESCRIPTION OF SHIP'S CHARACTERISTICS USED AS EXPLANATORY VARIABLES IN THESIS

	Characteristic	Symbol	Units	Definition
1.	Light Ship Weight	LSW	long tons	Weight of ship complete with all items of outfit, equipment, and machinery but excluding cargo, stores, crew, etc. Includes lead ballast for surface ships but not for submarines.
2.	Hull Weight	HULWGT	long tons	
3.	Propulsion Weight	PROWGT	long tons	These are the weights of the seven groups as described in Bureau of Ships Consolidated Index of Drawings, Materials, and Services Related to Construction Conversion.
4.	Electrical Weight	ELEWGT	long tons	
5.	Communication and Control weight	C+CWGT	long tons	
6.	Auxiliary Weight	AUXWGT	long tons	
7.	Outfitting Weight	OUTWGT	long tons	
8.	Armament Weight	ARMWGT	long tons	
9.	Complement	COMP	integer	Allowance for all officers and men on board.
10.	Maximum Shaft Horsepower	MAXSHP		Total power that can be applied continuously to the shafts under designed operating conditions. For subs this applies to surface operation.
11.	Range	RANGE	nautical miles	

	Characteristic	Symbol	Units	Definition
12.	Series	SERIES	integer	A number that represents the position of a ship in a series of similar ships. For example, DD 936 to DDG 30 have the same basic design except for missiles and constitute a series of 39 similar ships. DD 936 would be assigned series number 1 and DDG 30 would be assigned series number 39.
13.	DE or DEG	DEDEG	integer	1 = DE or DEG; zero otherwise.
14.	Prototype Dummy	PROTO	integer	1 = protytype; zero otherwise.
15.	Number of Generators	NO-GEN	integer	Number of ship service generators
16.	Power Loading Factor	PWRLD		Ratio of maximum shaft horsepower to full displacement.
17.	Award Date	AWARD	years	Last two digits of the year in which the contract was awarded.
18.	Generator Capacity	TKWCPY	kilowatts	Total maximum output of all generators.
19.	Propulsion and Auxiliary Weight	PRAXWT	long tons	Propulsion weight plus auxiliary weight.
20.	Prototype Light Ship Weight	PROLSW		For prototypes, this takes on the value of LSW. For non-prototypes, it has the value zero.
21.	Missile End Dummy	MS-END	integer	0 = no launchers; 1 = a launcher at one end of the ship; 2 = a launcher at each end of the ship.

	Characteristic	Symbol	Units	Definition
22.	Armament Weight-to-Light Ship Weight Ratio	AR/LSW		Ratio of armament weight to light ship weight.
23.	Engineering Weight	ENGWGT	long tons	Total weight of all propulsion, electrical, and auxiliary equipment.
24.	Payload Weight	PAYWGT	long tons	Total weight of all C+C, armament and outfitting equipment.
25.	Engine and Payload Weight	ENGPAY	long tons	ENGWGT + PAYWGT

APPENDIX B

BASIC CONTRACT COST CATEGORIES

Category No.	Category Name	Includes
1	Hull Structure	Shell plating and planking, logitudinal and transverse frames, decks, superstructure, armor, etc.
2	Propulsion	Boilers and energy converters, propulsion units, uptakes, propulsion control equipment, feedwater and condensate system, etc.
3	Electric Plant	Electric power generators, power distribution switchboards and cables, lighting systems, etc.
4	Communication and Control	Navigation equipment, interior communication equipment, fire control systems, radar systems, radio communications systems, sonar systems, etc.
5	Auxiliary	Heating, ventilating, air conditioning, plumbing, elevators, arresting gears, rudders, etc.
6	Outfit and Furnishing	Hull fittings, nonstructural bulkheads, paintings, equipment for work shops, furnishings for quarters, etc.
7	Armament	Guns and gun mount, ammunition handling and storage systems, other weapon systems handling and storage systems, etc.
8	Design and Engineering Services	Contract drawings, working drawings, technical manuals, lofting, mock-up and models, etc.
9	Construction	Staging, scaffolding and cribbing, launching, trials, cleaning ship, drydocking, etc.

END COST CATEGORIES

Miscellaneous End Cost	Disaster costs; cost of hull, mechanical and electrical changes; post-delivery costs; etc.
Weapons End Cost	Weapons costs after contractor delivery; missile, ASROC systems; etc.
Electronics End Cost	Electronics costs after contractor delivery; radar, NTDS, fire control systems; etc.

APPENDIX C

COMPUTER OUTPUT

MODEL B - 9 CERs - 36 OBSERVATIONS

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 1

SELECTION CARD 0 2 114
        LEAST SQUARES REGRESSION COEFICIENTS

    0.040621     0.138546
STD. ERROR OF ESTIMATE=        0.605304

        COVARIANCE MATRIX OF THE ESTIMATES

    0.782640-01 -0.378850-02
   -0.378850-02  0.210800-03
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.720130

MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 2

SELECTION CARD 0 3 23331
        LEAST SQUARES REGRESSION COEFICIENTS

   58.666710    -6.091645     0.000729
STD. ERROR OF ESTIMATE=        1.013978

        COVARIANCE MATRIX OF THE ESTIMATES

    0.301900 00 -0.457210-01  0.260750-05
   -0.457210-01  0.115510-01 -0.115920-05
    0.260750-05 -0.115920-05  0.158840-09
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.754360
```

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 3

SELECTION CARD 0 4 21625
        LEAST SQUARES REGRESSION COEFICIENTS

    0.148642    0.386163    0.261561
STD. ERROR OF ESTIMATE=        0.293750

        COVARIANCE MATRIX OF THE ESTIMATES

    0.366395-01 -0.174185-03 -0.101795-01
   -0.174185-03  0.102245-01 -0.397855-02
   -0.101795-01 -0.397855-02  0.459675-02
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.660927



MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 4

SELECTION CARD 0 5 21729
        LEAST SQUARES REGRESSION COEFICIENTS

   -0.903701    3.173884    0.347513
STD. ERROR OF ESTIMATE=        3.643863

        COVARIANCE MATRIX OF THE ESTIMATES

    0.259735 01 -0.121295 01 -0.779275 00
   -0.121295 01  0.698715 00  0.165625 00
   -0.779275 00  0.165625 00  0.200635 01
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.263167
```

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 5

SELECTION CARD C 6 118
        LEAST SQUARES REGRESSION COEFICIENTS

    0.026382    0.224177
STD. ERROR OF ESTIMATE=        0.570275

        COVARIANCE MATRIX OF THE ESTIMATES

    0.68175D-01 -0.59424D-02
   -0.59424D-02  0.597C7D-03
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.703815



MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 6

SELECTION CARD C 7 121
        LEAST SQUARES REGRESSION COEFICIENTS

    0.435838    0.038576
STD. ERROR OF ESTIMATE=        0.319834

        COVARIANCE MATRIX OF THE ESTIMATES

    0.20947D-01 -0.56669D-03
   -0.56669D-03  0.17737D-04
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.703134
```

```
MLLTIPLE REGRESSICN.....GENRAL
     SELECTICN..... 7

SELECTICN CARC O E 2282C
        LEAST SGLARES REGRESSICN CCEFICIENTS

   -2.608855     9.178115     3.764265
STD. ERRCR CF ESTIMATE=        3.793802

        CCVARIANCE MATRIX CF THE ESTIMATES

    C.23557D C1   C.924C4D CC -0.12052D 01
    C.924C4D CC   C.271C9D C1 -0.13240D 01
   -C.12C52D C1 -C.1324CD C1   0.992980 00
ADJUSTED MLLTIPLE CCRRELATICN CCEFICIENT 0.865614



MLLTIPLE REGRESSICN.....GENRAL
     SELECTICN..... 8

SELECTICN CARC O 9 22C23
        LEAST SGLARES REGRESSICN CCEFICIENTS

   -1.012537     0.653981     0.151729
STD. ERRCR CF ESTIMATE=        1.3C7084

        CCVARIANCE MATRIX CF THE ESTIMATES

    C.24949D CC -C.885C2D-C1 -0.12751D-02
   -C.885C2D-C1   C.41244D-C1 -0.17294D-03
   -0.12751D-C2 -C.17294D-C3   0.22420D-03
ADJUSTED MLLTIPLE CCRRELATICN CCEFICIENT 0.766887
```

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 9

SELECTION CARD CIC 3172223
        LEAST SQUARES REGRESSION COEFICIENTS

    3.137940     3.949713  -59.846808     0.059936
STD. ERROR OF ESTIMATE=        1.643151

        COVARIANCE MATRIX OF THE ESTIMATES

    0.115940 01 -0.125970 00 -0.12864D 02 -0.40064D-02
   -0.125970 00  0.157280 00 -0.19555D 01 -0.12869D-02
   -0.128640 02 -0.195550 01  0.24024D 03  0.52447D-01
   -0.400640-02 -0.128690-02  0.52447D-01  0.36985D-03

ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.776752
```

MATRIX OF RESIDUALS FOR SELECTIONS

CASE NO.	SELECTION NUMBER 1	2	3	4	5	6	7	8	9
1	-0.97461	1.09332	0.13241	-3.78724	-1.51320	0.55974	-0.98823	-3.52181	-2.71784
2	-0.55461	-1.84609	0.11241	-2.84973	-0.21320	0.19974	-1.21823	-0.26781	0.35812
3	-0.15828	-8.68408	0.47241	-3.29973	0.81939	0.10974	-1.15823	0.18219	-1.02188
4	-0.32745	1.82170	-0.04897	-0.87724	-0.61215	0.00748	-1.15521	3.12898	0.42541
5	-0.64352	-10.51763	-0.12145	-0.47060	-0.29942	-0.20721	-1.22342	0.10181	1.06182
6	0.53172	-9.36082	-0.11759	-0.45973	0.85939	-0.37412	-1.31635	-0.00781	-1.19838
7	-0.26004	-2.09861	0.22103	0.19027	0.87838	-0.33442	-1.13521	0.21451	0.63571
8	-0.46828	4.97080	0.05241	-0.86973	-0.65061	-0.38412	-3.40635	-0.22781	1.00162
9	0.22720	0.27929	-0.10987	-1.40233	-0.53444	-0.27148	-4.74106	-0.53473	-1.98358
10	-0.24828	-0.36339	0.37241	0.06027	0.28939	-0.21412	-4.78635	-0.48781	-0.74838
11	-0.37828	-5.24771	0.01241	0.32027	-0.02061	-0.04412	7.97365	-0.06781	1.33162
12	0.02172	-9.10375	0.31241	-0.99973	1.46939	-0.05412	-4.75635	-0.11781	0.60162
13	0.40937	-8.33254	-0.26531	-1.61060	0.14350	-0.23575	-1.71814	-0.14511	-0.88181
14	-0.43075	-7.88267	0.09330	-1.03407	-0.40839	-0.15881	-4.26521	-0.12549	-0.80073
15	0.74263	-5.82074	-0.09802	-0.21206	-0.21929	0.11268	0.90980	-1.15966	-0.48602
16	0.18263	-0.57696	0.04198	0.19545	0.03071	0.23268	0.43980	0.82281	0.16314
17	0.29263	-7.35824	0.11198	0.28545	-0.02929	0.37268	0.63980	1.19281	0.65314
18	0.64263	-0.36594	0.29198	0.51545	-0.07929	0.25268	0.44980	1.33281	0.42314
19	0.32070	-8.47721	-0.50779	-0.28858	-0.32135	-0.45433	1.36315	-0.26774	-2.00991
20	0.59070	-1.44299	-0.32779	0.18893	-0.17135	-0.28433	2.80315	1.23095	0.84121
21	-0.61975	9.23266	-0.29306	1.29665	-0.11935	0.44398	1.59030	-0.75318	-0.65711
22	-0.23756	1.16228	-0.07268	-0.17985	0.31078	0.08072	0.11638	-1.84896	1.28931
23	-0.56111	-0.78136	0.14732	-0.09234	0.27854	0.09499	0.60873	0.67922	1.54666
24	-0.08955	8.35373	0.12732	17.86766	-0.33474	-0.54652	0.36581	0.62970	4.00863
25	-0.27509	4.75800	-0.05724	-1.22668	0.19473	-0.02537	0.81752	0.98818	-2.60364
26	-0.22484	3.33332	-0.00724	1.30015	0.31922	-0.06124	1.09241	-3.31523	0.23002
27	-0.32719	2.74623	0.07890	0.01419	0.36990	0.03140	-4.09345	0.60738	-2.09494
28	0.40487	18.16768	0.44359	-1.64232	-0.26255	0.63327	5.97216	0.68018	3.57978
29	-0.09513	13.37890	-0.49641	-2.55232	-0.47255	0.02327	-2.93784	0.45018	-0.89022
30	0.21293	17.53721	-0.35306	-1.48113	0.00356	0.56190	10.70899	2.51348	0.18506
31	0.32889	-3.13824	-0.56375	-2.81404	0.17787	0.13641	5.60256	-0.51183	1.98930
32	1.65293	5.51085	-0.60306	-3.32362	0.13356	0.36190	-4.85101	-0.51643	-1.31276
33	1.63293	5.67560	0.30694	-1.46362	0.71356	0.01190	-4.72101	-0.84643	1.36724
34	-0.53580	1.47415	0.13316	4.22931	-0.55233	0.12701	5.53688	0.86243	1.07835
35	-0.94995	-4.17831	0.14446	3.16249	-0.79560	0.00890	0.64038	-0.57712	-2.01522
36	0.16559	-3.89846	0.43446	3.31075	0.61785	-0.71302	0.84038	-0.31712	-1.35070

MATRIX OF CORRELATIONS BETWEEN SELECTIONS

SELECTION NO.	1	2	3	4	5	6	7	8	9
1	1.000000	0.083952	-0.226913	-0.141016	0.359145	0.021498	-0.094030	0.105750	0.102316
2	0.083952	1.000000	-0.109229	0.105069	-0.238084	0.424064	0.262607	0.160298	0.252176
3	-0.226913	-0.109229	1.000000	0.194853	0.234251	-0.050094	-0.158535	-0.023243	0.179807
4	-0.141016	0.105069	0.194853	1.000000	-0.071872	-0.405310	0.139125	0.115054	0.401507
5	0.359145	-0.238084	0.234251	-0.071872	1.000000	-0.220777	-0.152113	0.057277	0.101191
6	0.021498	0.424064	-0.050094	-0.405310	-0.220777	1.000000	0.322134	0.021004	0.058955
7	-0.094030	0.262607	-0.158535	0.139125	-0.152113	0.322134	1.000000	0.251701	0.362376
8	0.105750	0.160298	-0.023243	0.115054	0.057277	0.021004	0.251701	1.000000	0.234359
9	0.102316	0.252176	0.179807	0.401507	0.101191	0.058955	0.362376	0.234359	1.000000

JOINT GENERALIZED LEAST SQUARES REGRESSION OUTPUT

COVARIANCE MATRIX OF THE JOINT ESTIMATES

```
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 0.152080D-01 -0.846190D-03 -0.169410 01  0.501230 00 -0.500087D-04 -0.139300D-01  0.139040D-02  0.359630D-02  0.139040D-02 -0.235470D-01
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-0.101610-03
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JOINT LEAST SQUARES ESTIMATES

SELECTION NO.	COEFICIENTS			
	CONSTANT	B1	B2	B3
1	C.083120	0.136181		
2	58.704516	-5.925497	0.000697	
3	C.031010	0.352351	C.309889	
4	-C.620830	2.963455	0.627307	
5	-C.005553	0.227385		
6	0.549161	0.035029		
7	-2.576043	8.845534	3.859541	
8	-C.926037	0.638683	C.144506	
9	2.520405	3.662766	-42.152667	0.C51540

TABLE OF PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	53.343975	38.965281	27.270000	24.586588
2	46.997922	32.725161	28.390000	18.452401
3	47.295003	33.020052	26.400000	18.745101
4	61.820072	47.638389	51.610000	33.456706
5	56.298039	42.233032	40.040000	28.173025
6	55.445687	41.384644	39.150000	27.323601
7	54.759366	40.701178	41.020000	26.642991
8	55.445687	41.384644	38.200000	27.323601
9	56.566354	42.505990	34.570000	28.445627
10	55.445687	41.384644	35.910000	27.323601
11	55.445687	41.384644	50.980000	27.323601
12	55.445687	41.384644	37.340000	27.323601
13	56.425623	42.367731	37.320000	28.309840
14	56.314486	42.257164	33.690000	28.199842
15	26.982126	12.552976	11.320000	-1.876175
16	23.871400	9.500342	11.510000	-4.870715
17	23.871400	9.500342	12.080000	-4.870715
18	23.871400	9.500342	12.490000	-4.870715
19	25.638338	11.070105	8.700000	-3.498128
20	22.256424	7.756960	12.930000	-6.744504
21	30.654329	16.236835	18.490000	1.819341
22	40.509800	26.250017	25.090000	11.990233
23	34.528324	20.390131	23.990000	6.251937
24	34.540087	20.406466	44.700000	6.272846
25	35.180142	21.035734	20.250000	6.891326
26	51.356770	36.968139	28.490000	22.579509
27	45.738793	31.497752	27.270000	17.256711
28	63.377046	49.206398	67.590000	35.035749
29	63.394785	49.224009	46.230000	35.053232
30	89.087878	74.517103	93.900000	59.946327
31	82.519059	68.023626	75.310000	53.528193
32	78.410976	64.069823	59.300000	49.728670
33	78.410976	64.069823	62.900000	49.728670
34	82.366152	67.784101	80.040000	53.202050
35	72.664380	58.251049	59.500000	43.837718
36	72.721310	58.304188	62.170000	43.887065

MODEL B

JOINT GERERALIZED LEAST SQUARES

PREDICTION INTERVAL VALUES

LEAST SQUARES COVARIANCE MATRIX

0.739160-01	-0.357800-02	-0.525890-01	0.145960-01	-0.136600-05	-0.919190-02	0.312010-02	0.119450-02	-0.52045D-01	0.26448D-01
-0.357800-02	0.199090-03	0.347150-02	-0.812180-03	0.760050-07	0.453430-03	-0.173610-03	-0.664670-04	0.244860-02	-0.147160-02
-0.525890-01	0.347150-02	0.149870 02	-0.226980 01	0.129440-03	0.437340-01	-0.814380-02	-0.116150-01	-0.217730 00	0.189470 00
0.145960-01	-0.812180-03	-0.226980 01	0.573430 00	-0.575490-04	-0.118440-01	0.306800-02	0.231450-02	0.759440-01	-0.470420-01
-0.136600-05	0.760050-07	0.129440-03	-0.575490-04	0.788540-08	0.112550-05	-0.393700-06	-0.159660-03	-0.179650-06	0.465110-05
-0.919190-02	0.453430-03	0.437340-01	-0.118440-01	0.112550-05	0.337690-01	-0.159660-03	-0.933030-02	0.309470-01	-0.158510-01
0.312010-02	-0.173610-03	-0.814380-02	0.306800-02	-0.393700-06	-0.159660-03	0.937170-02	-0.364700-02	-0.199410-01	0.110900-01
0.119450-02	-0.664670-04	-0.116150-01	0.231450-02	-0.179650-06	-0.933030-02	-0.364700-02	0.421360-02	0.233620-03	0.343990-03
-0.520450-01	0.244860-02	-0.217730 00	0.759440-01	-0.783910-05	0.309470-01	-0.199410-01	0.233630-03	0.238050 01	-0.111180 01
0.264480-01	-0.147160-02	0.189470 00	-0.470420-01	0.465110-05	-0.158510-01	0.110900-01	0.343990-03	-0.111180 01	0.640490 00
-0.135040-02	0.751370-04	-0.108800 00	0.117180-01	0.162820-06	0.376330-02	0.538260-02	-0.324160-02	-0.714330 00	0.151620 00
0.246310-01	-0.118960-02	0.143530 00	-0.389500-01	0.357550-05	0.939840-02	-0.254060-02	-0.149290-02	-0.237920-01	0.120530-01
-0.214810-02	0.119520-03	-0.170530-01	0.391360-02	-0.359260-06	-0.842420-03	0.255270-03	0.150010-03	0.200270-02	-0.121110-02
0.797040-03	-0.382730-04	-0.107940 00	0.354150-01	-0.368470-05	-0.891510-03	0.453460-03	0.502930-04	-0.812360-01	0.409260-01
-0.215290-04	0.119790-05	0.419710-02	-0.110840-02	0.115330-06	0.240960-04	-0.141930-04	-0.157410-05	0.216050-02	-0.128090-02
-0.335900-01	0.155850-02	-0.923140 00	0.234140 00	-0.195170-04	-0.361810-01	0.575840-02	0.715590-02	0.219880 00	-0.105830 00
0.408050-03	-0.227610-04	-0.253250 00	0.115470-01	0.329330-05	-0.603740-02	-0.511860-02	0.381410-02	-0.257530-01	0.133870-01
0.127320-01	-0.708430-03	0.595100 00	-0.111400 00	0.787350-05	0.165580-01	-0.947160-03	-0.455310-02	-0.699750-01	0.441760-01
0.132180-01	-0.615140-03	-0.171500 00	0.485200-01	-0.445640-05	-0.171080-02	0.434520-03	0.269540-03	0.705570-01	-0.329540-01
-0.499050-02	0.277690-03	0.997500-01	-0.222280-01	0.197840-05	0.689240-03	-0.166550-03	-0.139410-03	-0.227880-01	0.142460-01
-0.261330-04	0.145410-05	-0.789220-03	-0.185910-04	0.203740-07	-0.226200-05	-0.982860-05	0.454760-05	-0.950580-03	0.264080-03
0.225420-01	-0.113250-02	-0.789730 00	0.157700 00	-0.106130-04	0.275130-01	-0.226160-02	-0.664670-02	0.370920 00	-0.186850 00
-0.763590-02	0.424880-03	0.137430 00	-0.404150-01	0.453970-05	-0.452450-02	0.498790-02	-0.620160-03	-0.181860 00	0.110710 00
-0.111870 00	0.622500-02	0.970110 01	-0.134890 01	0.418050-04	-0.267990 00	-0.990640-01	0.118780 00	0.290470 00	-0.617350-01
-0.287920-04	0.160210-05	0.789580-04	-0.199520-03	0.352700-07	-0.237220-04	0.537400-04	-0.141270-04	-0.336390-02	0.714930-03

-0.135040-02	0.246310-01	-0.214810-02	0.797040-03	-0.215290-04	-0.335900-01	0.408050-03	0.127320-01	0.132180-01	-0.499050-02
0.751370-04	-0.118960-02	0.119520-03	-0.382730-04	0.119790-05	0.155850-02	-0.227610-04	-0.708430-03	-0.615140-03	0.277690-03
-0.108800 00	0.143530 00	-0.170530-01	-0.107940 00	0.419710-02	-0.923140 00	-0.253250 00	0.595100 00	-0.171500 00	0.997500-01
0.117180-01	-0.389500-01	0.391360-02	0.354150-01	-0.110840-02	0.234140 00	0.115470-01	-0.111400 00	0.485200-01	-0.222280-01
0.162820-06	0.357550-05	-0.359260-06	-0.368470-05	0.115330-06	-0.195170-04	0.329330-05	0.787350-05	-0.445640-05	0.197840-05
0.376330-02	0.939840-02	-0.842420-03	-0.891510-03	0.240960-04	-0.361810-01	-0.603740-02	0.165580-01	-0.171080-02	0.689240-03
0.538260-02	-0.254060-02	0.255270-03	0.453460-03	-0.141930-04	0.575840-02	-0.511860-02	-0.947160-03	0.434520-03	-0.166550-03
-0.324160-02	-0.149290-02	0.150010-03	0.502930-04	-0.157410-05	0.715590-02	0.381410-02	-0.455310-02	0.269540-03	-0.139410-03
-0.714330 00	-0.237920-01	0.200270-02	-0.812360-01	0.216050-02	0.219880 00	-0.257530-01	-0.699750-01	0.705570-01	-0.227880-01
0.151620 00	0.120530-01	-0.121110-02	0.409260-01	-0.128090-02	-0.105830 00	0.133870-01	0.441760-01	-0.329540-01	0.142460-01
0.183510 01	-0.110870-02	0.111400-03	0.163190-02	-0.510770-04	0.260890-01	0.132340-01	-0.163770-01	-0.540360-02	-0.439130-02
-0.111870-02	0.643980-01	-0.561220-02	0.743160-02	0.199540-03	-0.516770-01	-0.111860-02	0.201980-01	0.670430-02	-0.254330-02
0.111400-03	-0.561220-02	0.563900-03	0.640560-03	-0.200490-04	0.433710-02	0.111390-03	-0.202950-02	-0.562750-03	0.255550-03
0.163190-02	0.743160-02	0.640560-03	0.197840-01	-0.535210-03	0.533720-01	-0.741560-02	-0.174190-01	0.128250-02	-0.469030-03
-0.510770-04	0.199540-03	-0.200490-04	-0.535210-03	0.167510-04	-0.135430-02	0.232100-03	0.545180-03	-0.330370-04	0.146800-04
0.260890-01	-0.516770-01	0.433710-02	0.533720-01	-0.135430-02	0.215940 01	0.847040 00	-0.110470 01	0.161810 00	-0.599830-01
0.132340-01	-0.111860-02	0.111390-03	-0.741560-02	0.232100-03	0.847040 00	0.248500 01	-0.121370 01	-0.121670-02	0.165020-03
-0.163770-01	0.201980-01	-0.202950-02	-0.174190-01	0.545180-03	-0.110470 01	-0.121370 01	0.910230 00	-0.593320-01	0.276120-01
-0.540360-02	0.670430-02	-0.562750-03	0.128250-02	-0.330370-04	0.161810 00	-0.121670-02	-0.593320-01	0.228700 00	-0.811260-01
-0.439130-02	-0.254330-02	0.255550-03	-0.469030-03	0.146800-04	-0.599830-01	-0.165020-03	0.276120-01	-0.811260-01	0.378070-01
0.203120-02	-0.879430-05	0.883630-06	-0.469810-05	0.147040-06	0.729250-04	0.213940-03	-0.104450-03	-0.116880-02	-0.158530-03
0.105480-01	0.219710-01	-0.196510-02	0.563200-02	-0.151590-03	0.482890 00	0.109860 00	-0.232280 00	0.100800 00	-0.395910-01
-0.150860-01	-0.656190-02	0.659320-03	-0.255630-02	0.800100-04	-0.927720-01	0.339810-01	0.313460-01	-0.229570-01	0.107170-01
-0.747860 00	-0.127730 00	0.128340-01	-0.750170-02	0.234790-03	-0.407780 01	-0.251560 01	0.270810 01	-0.709660 00	0.330720 00
0.866060-02	-0.241960-04	-0.243110-05	-0.887220-05	0.277690-06	-0.442180-03	-0.251280-03	0.286520-03	-0.417660-03	-0.937650-05

-0.261330-04	0.225420-01	-0.763590-02	-0.111870 00	-0.287920-04
0.145410-05	-0.113250-02	0.424880-03	0.622500-02	0.160210-05
-0.789220-03	-0.789730 00	0.137430 00	0.970110 01	0.789580-04
-0.185910-04	0.157700 00	-0.404150-01	-0.134890 01	-0.199520-03
0.203740-07	-0.106130-04	0.453970-05	0.418050-04	0.352700-07
-0.226200-05	0.275130-01	-0.452450-02	-0.267990 00	-0.237220-04
-0.982860-05	-0.226160-02	0.498790-02	-0.990640-01	0.537400-04
0.454760-05	-0.664670-02	-0.620160-03	0.118780 00	-0.141270-04
-0.950580-03	0.370920 00	-0.181860 00	0.290470 00	-0.336390-02
0.264080-03	-0.186850 00	0.110710 00	-0.617350-01	0.714930-03
0.203120-02	0.105480-01	-0.150860-01	-0.747860 00	0.866060-02
-0.879430-05	0.219710-01	-0.656190-02	-0.127730 00	-0.241960-04
0.883630-06	-0.196510-02	0.659320-03	0.128340-01	-0.243110-05
-0.469810-05	0.563200-02	-0.255630-02	-0.750170-02	-0.887220-05
0.147040-06	-0.151590-03	0.800100-04	0.234790-03	0.277690-06
0.729250-04	0.482890 00	-0.927720-01	-0.407780 01	-0.442180-03
0.213940-03	0.109860 00	0.339810-01	-0.251560 01	-0.251280-03
-0.104450-03	-0.232280 00	0.313460-01	0.270810 01	0.286520-03
-0.116880-02	0.100800 00	-0.229570-01	-0.709660 00	-0.417660-03
-0.158530-03	-0.395910-01	0.107170-01	0.330720 00	-0.937650-05
0.205520-03	-0.271780-03	-0.449370-04	-0.138670-02	0.594710-04
-0.271780-03	0.103060 01	-0.111970 00	-0.114350 02	-0.356130-02
-0.449370-04	-0.111970 00	0.139800 00	-0.173820 01	-0.114390-02
-0.138670-02	-0.114350 02	-0.173820 01	0.213550 03	0.466200-01
0.594710-04	-0.356130-02	-0.114390-02	0.466200-01	0.328760-03

TABLE OF LS CORRELATED PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	52.865238	38.269414	27.270000	23.673590
2	46.266816	31.861932	28.390000	17.457048
3	46.564809	32.156748	28.400000	17.748688
4	61.815729	47.526568	51.610000	33.237407
5	56.044064	41.969404	40.040000	27.894744
6	55.116949	41.040287	39.150000	26.963624
7	54.440909	40.367181	41.020000	26.293453
8	55.116949	41.040287	38.200000	26.963624
9	56.332498	42.257703	34.570000	28.182909
10	55.116949	41.040287	35.910000	26.963624
11	55.116949	41.040287	50.980000	26.963624
12	55.116949	41.040287	37.340000	26.963624
13	56.223151	42.151262	37.320000	28.079373
14	56.128873	42.057563	33.690000	27.986254
15	26.202169	11.688001	11.320000	-2.826167
16	23.137449	8.712043	11.510000	-5.713362
17	23.137449	8.712043	12.080000	-5.713362
18	23.137449	8.712043	12.490000	-5.713362
19	25.525223	10.853086	8.700000	-3.819051
20	22.184774	7.605762	12.930000	-6.973249
21	30.701544	16.185230	18.490000	1.668917
22	40.942925	26.583046	25.090000	12.223167
23	34.739514	20.566869	23.990000	6.394225
24	34.720124	20.552871	44.700000	6.385617
25	35.518926	21.338761	20.250000	7.158596
26	52.282515	37.716865	38.490000	23.151215
27	46.427530	32.083912	27.270000	17.740294
28	63.421959	49.222259	67.590000	35.022559
29	63.439720	49.239859	46.230000	35.039998
30	90.414639	75.620819	93.900000	60.827000
31	83.443143	68.867100	75.310000	54.291057
32	79.079522	64.671006	59.300000	50.262491
33	79.079522	64.671006	62.900000	50.262491
34	84.022361	69.223926	80.040000	54.425492
35	73.769632	59.301116	59.500000	44.832599
36	73.834053	59.361273	62.170000	44.888493

TABLE OF LS (INDEPEND) PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	50.588425	38.268265	27.269989	25.948090
2	43.976730	31.860794	28.389999	19.744843
3	44.291870	32.155594	28.399994	20.019302
4	59.434250	47.525436	51.609985	35.616608
5	53.655869	41.968307	40.039993	30.280731
6	52.730240	41.039185	39.149994	29.348114
7	52.055588	40.366074	41.019989	28.676544
8	52.730240	41.039185	38.199997	29.348114
9	53.944000	42.256592	34.569992	30.569168
10	52.730240	41.039185	35.909988	29.348114
11	52.730240	41.039185	50.979996	29.348114
12	52.730240	41.039185	37.339996	29.348114
13	53.835571	42.150146	37.319992	30.464706
14	53.741562	42.056442	33.689987	30.371307
15	23.676498	11.687059	11.320000	-0.302390
16	20.603622	8.711105	11.509999	-3.181420
17	20.603622	8.711105	12.080000	-3.181420
18	20.603622	8.711105	12.490000	-3.181420
19	22.893280	10.851655	8.700000	-1.189982
20	19.552795	7.604342	12.929999	-4.344112
21	28.102158	16.184311	18.489990	4.266452
22	38.430557	26.582047	25.089996	14.733521
23	32.233276	20.565872	23.989990	8.898454
24	32.214645	20.551880	44.699997	8.889112
25	33.016861	21.337753	20.250000	9.658632
26	49.826096	37.715866	38.489990	25.605621
27	43.976547	32.082932	27.269989	20.189301
28	61.001038	49.220993	67.589996	37.440933
29	61.019516	49.238586	46.229996	37.457642
30	87.900284	75.618988	93.899994	63.337677
31	80.925568	68.865265	75.309998	56.804947
32	76.597549	64.669189	59.299988	52.740814
33	76.597549	64.669189	62.899994	52.740814
34	81.463455	69.222061	80.039993	56.980652
35	71.255280	59.299240	59.500000	47.343185
36	71.319580	59.359375	62.169998	47.399155

MODEL C - 9 CERs - 27 OBSERVATIONS

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MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 1

SELECTION CARD 0 2 114
        LEAST SQUARES REGRESSION COEFICIENTS

    0.552360     0.097619
STD. ERROR OF ESTIMATE=        0.396638

        COVARIANCE MATRIX OF THE ESTIMATES

    0.52175D-01 -0.30789D-02
   -0.30789D-02  0.20454D-03
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.636825



MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 2

SELECTION CARD 0 3 133
        LEAST SQUARES REGRESSION COEFICIENTS

   38.066134    -0.492150
STD. ERROR OF ESTIMATE=        0.714668

        COVARIANCE MATRIX OF THE ESTIMATES

    0.14036D 00 -0.13859D-01
   -0.13859D-01  0.15817D-02
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.827510
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MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 3

SELECTION CARD C 4 21625
        LEAST SQUARES REGRESSION COEFICIENTS

   -C.C15067     C.657699     C.23C566
STD. ERROR OF ESTIMATE=        0.218241

         COVARIANCE MATRIX OF THE ESTIMATES

    C.28C98D-C1 -C.13865D-C1 -0.37943D-02
   -0.13865D-C1  C.3117CD-C1 -0.58711D-02
   -0.37943D-C2 -C.58711D-C2  0.314C6D-02
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.739007


MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 4

SELECTION CARD 0 5 21728
        LEAST SQUARES REGRESSION COEFICIENTS

   -C.445966     3.429CC6    -1.284769
STD. ERROR OF ESTIMATE=        3.851717

         COVARIANCE MATRIX OF THE ESTIMATES

    C.47734D C1 -C.35441D C1  0.12189D 01
   -0.35441D C1  C.33823D C1 -0.21745D 01
    C.12189D C1 -C.21745D C1  0.35989D 01
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.C66788
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MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 5

SELECTION CARD 0 6 118
        LEAST SQUARES REGRESSION COEFICIENTS

   -C.CCC657     C.229295
STD. ERROR OF ESTIMATE=        0.595721

        COVARIANCE MATRIX OF THE ESTIMATES

    0.1C195D 00 -C.1051 9D-C1
   -0.1C519D-C1  C.1245 9D-C2
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.613091


MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 6

SELECTION CARD C 7 119
        LEAST SQUARES REGRESSION COEFICIENTS

    0.571337     C.CC3315
STD. ERROR OF ESTIMATE=        0.253202

        COVARIANCE MATRIX OF THE ESTIMATES

    0.38726D-C1 -C.14722D-C3
   -0.14722D-C3  C.5962CD-C6
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.4C1430
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```
MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 7

SELECTION CARD C $ 2282C
        LEAST SQUARES REGRESSION COEFICIENTS

   -1.CCCC13     7.991636     2.813978
STD. ERROR OF ESTIMATE=        2.504982

        COVARIANCE MATRIX OF THE ESTIMATES

    C.12C29D C1   C.29147D CC -0.63612D 00
    C.29147D CC   C.16556D C1 -0.62345D 00
   -0.63612D CO -C.62345D CC   0.53616D 00
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.851478



MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 8

SELECTION CARD C $ 22023
        LEAST SQUARES REGRESSION COEFICIENTS

   -C.306542     C.406525     0.094358
STD. ERROR OF ESTIMATE=        1.237198

        COVARIANCE MATRIX OF THE ESTIMATES

    0.32431D CO -C.13724D CC -0.47438D-02
   -0.13724D CC   C.75326D-C1  0.96585D-03
   -0.47438D-02   C.96585D-C3  0.51858D-03
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.379376
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MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 9

SELECTION CARD OIC 3172223
        LEAST SQUARES REGRESSION COEFICIENTS

    3.741737     3.940985  -68.643523     0.038777
STD. ERROR OF ESTIMATE=        1.560890

        COVARIANCE MATRIX OF THE ESTIMATES

    0.12337D 01 -C.26698D CC -0.11330D 02 -C.912C7D-C2
   -C.26698D CO  C.41816D CC -0.45499D 01 -0.14559D-C2
   -C.1133CD 02 -C.45499D C1  0.26394D 03  C.95685D-C1
   -0.912C7D-C2 -C.14559D-C2  0.95685D-01  C.84052D-C3
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.617770
```

MATRIX OF RESIDUALS FOR SELECTIONS

CASE NO.	SELECTION NUMBER 1	2	3	4	5	6	7	8	9
1	-0.66986	7.77760	0.08068	-4.31331	-1.54477	0.52741	-0.12633	-1.94190	-1.90289
2	-0.24986	4.83818	0.06068	-3.72331	-0.24477	0.16741	-0.35633	-0.33042	0.56729
3	0.13869	-1.50078	0.42068	-4.17331	0.78880	0.07741	-0.29633	0.11958	-0.81271
4	-0.06322	7.51324	-0.07355	-0.11855	-0.64095	0.17692	0.81715	4.85604	1.20605
5	-0.34205	-4.82610	-0.17590	-0.09003	-0.33273	0.06981	0.79645	0.03177	1.17616
6	0.82869	-3.66928	-0.16932	-0.04855	0.82680	-0.27259	0.73203	-0.07042	-1.07542
7	-0.00686	3.59293	0.19645	0.60145	0.85096	-0.17308	0.82715	0.13210	0.75340
8	-0.17131	10.66234	0.00068	-0.45855	-0.68120	-0.28259	-1.35797	-0.29042	1.12458
9	0.54914	5.97082	-0.15617	-1.02432	-0.56922	-0.05313	-2.71169	-0.60229	-1.87444
10	0.04869	5.32815	0.32068	0.47145	0.25880	-0.11259	-2.73797	-0.55042	-0.62542
11	-0.08131	0.44383	-0.03932	0.73145	-0.05120	0.05741	10.02203	-0.13042	1.45458
12	0.31869	-3.41221	0.26068	-0.58855	1.43880	0.04741	-2.70797	-0.18042	0.72458
13	0.72476	-2.64101	-0.32247	-1.23003	0.10912	0.02339	0.28273	-0.22010	-0.78859
14	-0.13010	-2.19113	0.06330	-0.65861	-0.44190	0.11992	-2.29285	-0.20790	-0.71173
15	0.51943	-7.83075	-0.02438	-0.43709	-0.21149	-0.00870	0.07069	-0.91940	-0.26279
16	-0.04057	-2.13347	0.11562	-0.37709	0.03851	0.11130	-0.39931	0.31726	0.11131
17	0.06943	-8.91475	0.18562	-0.28709	-0.02149	0.25130	-0.19931	0.68726	0.60131
18	0.41943	-1.94245	0.36562	-0.05709	-0.07149	0.13130	-0.38931	0.82726	0.37131
19	0.06557	2.65915	-0.35971	-0.50851	-0.31315	-0.51223	0.13442	-0.08877	-2.06221
20	0.33557	9.69337	-0.17971	-0.37851	-0.16315	-0.34223	1.57442	0.62393	0.49803
21	-0.73000	-5.71978	-0.39479	0.96702	-0.12046	0.51140	0.55163	-0.25451	-0.57709
22	-0.21848	-4.32318	-0.14296	-0.68041	0.30415	0.07355	-0.49466	-0.75816	1.61055
23	-0.53507	-6.26681	0.07704	-0.94041	0.27190	0.02382	0.00720	0.23563	1.31116
24	-0.07579	2.86827	0.05704	17.01959	-0.34122	-0.52672	-0.20722	0.19343	3.78413
25	-0.25969	-0.72746	-0.11665	-2.11047	0.18687	0.06291	0.13045	0.52221	-2.86651
26	-0.17998	-2.33081	-0.06665	2.02568	0.31145	-0.10618	1.72486	-2.19181	0.57566
27	-0.26391	-2.91790	0.01677	0.38710	0.36101	-0.04259	-3.40398	0.19091	-2.30934

MATRIX OF CORRELATIONS BETWEEN SELECTIONS

SELECTION NO.	1	2	3	4	5	6	7	8	9
1	1.000000	-0.035010	0.029758	0.001132	0.328452	-0.418906	-0.060438	0.088406	-0.104664
2	-0.035010	1.000000	0.004470	0.009239	-0.449550	-0.291248	-0.012811	0.148321	0.000102
3	0.029758	0.004470	1.000000	-0.040627	0.282828	0.122766	-0.229566	0.033536	0.187222
4	0.001132	0.009239	-0.040627	1.000000	-0.009807	-0.499669	0.062083	0.063300	0.577040
5	0.328492	-0.449550	0.282828	-0.009807	1.000000	-0.206360	-0.058409	-0.018841	0.049356
6	-0.418906	-0.291248	0.122766	-0.499669	-0.206360	1.000000	0.018820	0.006415	-0.192862
7	-0.060438	-0.012811	-0.229566	0.062083	-0.058409	0.018820	1.000000	0.049265	0.305964
8	0.088406	0.148321	0.033536	0.063300	-0.018841	0.006415	0.049265	1.000000	0.195850
9	-0.104664	0.000102	0.187222	0.577040	0.049356	-0.192862	0.305964	0.195850	1.000000

JOINT GENERALIZED LEAST SQUARES REGRESSION OUTPUT

COVARIANCE MATRIX OF THE JOINT ESTIMATES

0.442110-01	-0.257860-02	0.238220-01	-0.301770-02	0.199720-02	0.805570-03	-0.872480-03	-0.635030-02	-0.931540-04	0.135720-01
-0.257860-02	0.171300-03	-0.175000-02	0.200470-03	-0.126930-03	-0.535150-04	0.575590-04	0.425720-03	0.618820-05	-0.901620-03
0.238220-01	-0.175000-02	0.589250 01	-0.553940 00	0.432590-02	0.146370-01	-0.614500-02	-0.505640-01	0.533790-01	-0.318040-01
-0.301770-02	0.200470-03	-0.553940 00	0.632180-01	-0.474240-03	-0.167040-02	0.701290-03	0.652150-02	-0.609180-02	0.362960-02
0.199720-02	-0.126930-03	0.433590-02	-0.474240-03	0.223980-01	-0.115200-01	0.215990-01	-0.346080-02	-0.234450-02	0.655090-03
0.805570-03	-0.535150-04	0.146370-01	-0.167040-02	-0.115200-01	0.215990-01	-0.346080-02	-0.234450-02	0.247700-02	-0.211370-02
-0.872480-03	0.575590-04	-0.614500-02	0.701290-03	-0.313670-02	-0.346080-02	0.213720-02	0.234150-02	-0.189380-02	0.475990-03
-0.635030-02	0.425720-03	-0.505640-01	0.652150-02	-0.600627D-02	-0.234450-02	0.234150-02	0.378120 01	-0.260380 01	0.501640 00
-0.931540-04	0.618820-05	0.533790-01	-0.609180-02	0.399160-02	0.247700-02	-0.189380-02	-0.260380 01	0.223010 01	-0.879210 00
0.135720-01	-0.901620-03	-0.318040-01	0.362960-02	-0.655090-03	-0.211370-02	0.475990-03	0.501840 00	-0.879210 00	0.143640 01
0.174120-01	-0.979650-03	0.162760 00	-0.243430-01	0.137860-01	-0.356600-02	-0.280440-02	-0.106910-01	0.840610-03	0.182630-01
-0.174710-02	0.116600-03	-0.252650-01	0.288330-02	-0.148650-02	0.424750-03	0.332180-03	0.117670-02	-0.995600-04	-0.216330-02
-0.124870-01	0.733690-03	0.545580-01	-0.781470-02	0.198790-02	-0.306720-02	0.452020-03	-0.159250 00	0.108890 00	-0.101780-01
0.447280-04	-0.297130-05	-0.277310-03	0.316480-04	-0.712760-05	0.124210-04	-0.183060-05	0.578770-03	-0.441000-03	0.412180-04
-0.114370-01	0.625760-03	-0.233500-01	0.198760-02	-0.330650-01	0.841170-02	0.641760-02	0.726970-01	-0.266120-01	-0.337580-01
-0.856370-03	0.568690-04	-0.714170-02	0.815030-03	-0.611090-03	0.190570-02	-0.434090-03	-0.888340-01	0.128340 00	-0.177330 00
0.563020-02	-0.374020-03	0.119420-01	-0.136290-02	0.167370-01	-0.534230-02	0.355530-02	-0.549990-02	-0.201470-01	0.682230-01
0.111970-01	-0.646980-03	-0.837590-01	0.134310-01	-0.151180-02	-0.225200-02	0.356840-03	0.438480-01	-0.227530-01	-0.625640-02
-0.556840-02	0.369910-03	0.689190-01	-0.786520-02	-0.653580-03	0.128510-02	-0.203300-03	-0.180750-01	0.115260-01	0.504660-02
-0.258500-05	0.171720-06	-0.451360-03	0.515110-04	-0.428240-06	0.135040-05	-0.305140-06	-0.356990-03	0.445340-03	-0.426850-03
-0.206270-01	0.122670-02	-0.132060-01	0.151060-02	-0.244450-01	-0.156770-01	-0.154350-02	0.809730 00	-0.491000 00	-0.652840-01
0.921760-02	-0.612330-03	0.198440-01	-0.226470-02	-0.941750-02	0.173330-01	-0.272240-02	-0.533810 00	0.406600 00	-0.376250-01
0.944300-01	-0.627310-02	-0.210670 00	0.240430-01	-0.145020 00	-0.127770 00	0.881830-01	0.454310 00	-0.101580 01	0.192020 01
-0.132450-04	0.300760-04	0.343240-05	-0.504670-04	0.594860-04	-0.357860-05	-0.422500-03	0.726050-03	-0.116940-02	

0.174120-01	-0.174710-02	-0.124870-01	0.447280-04	-0.114370-01	-0.856370-03	0.563020-02	0.111970-01	-0.556840-02	-0.258500-05
-0.979650-03	0.116600-03	0.733690-03	-0.297130-05	0.625760-03	0.568690-04	-0.374020-03	-0.646980-03	0.369910-03	0.171720-06
0.162760 00	-0.252650-01	0.545580-01	-0.277310-03	-0.233500-01	-0.714170-02	0.119420-01	-0.837590-01	0.689190-01	-0.451360-03
-0.243430-01	0.288330-02	-0.781470-02	0.316480-04	0.198760-02	0.815030-03	-0.136290-02	0.134310-01	-0.786520-02	0.515110-04
0.137860-01	-0.148650-02	0.198790-02	-0.712760-05	-0.330650-01	-0.611090-03	0.167370-01	-0.151180-02	-0.653580-03	-0.428240-06
-0.356600-02	0.424750-03	-0.306720-02	0.124210-04	0.841170-02	0.190570-02	-0.534230-02	-0.225200-02	0.128510-02	0.135040-05
-0.280440-02	0.332180-03	0.452020-03	-0.183060-05	0.641760-02	-0.434090-03	0.355530-02	0.356840-03	-0.203300-03	-0.305140-06
-0.106910-01	0.117670-02	-0.159250 00	0.578770-03	0.726970-01	-0.888340-01	-0.549990-02	0.438480-01	-0.180750-01	-0.356990-03
0.840610-03	-0.995600-04	0.108890 00	-0.441000-03	-0.266120-01	0.128340 00	-0.201470-01	-0.227530-01	0.115260-01	0.445340-03
0.182630-01	-0.216330-02	-0.101780-01	0.412180-04	-0.337580-01	-0.177330 00	0.682230-01	-0.625640-02	0.504660-02	-0.426850-03
0.803910-01	-0.808060-02	-0.100910-01	0.365430-04	-0.167570-01	-0.227870-02	0.854700-02	0.218050-02	-0.150630-02	-0.282900-05
-0.808060-02	0.957120-03	0.106880-02	-0.432840-05	0.169790-02	0.269910-03	-0.101240-02	-0.313540-03	0.178410-03	0.335090-06
-0.100910-01	0.106880-02	0.278560-01	-0.103910-03	0.341980-02	0.123450-02	-0.206900-02	0.915210-03	-0.337240-03	-0.445450-04
0.365430-04	-0.432840-05	-0.103910-03	0.420810-06	-0.122250-04	-0.459540-05	0.837900-05	-0.344520-05	0.136610-05	0.180400-06
-0.167570-01	0.169790-02	0.341980-02	-0.122250-04	0.105040 01	0.218800 00	-0.543540 00	0.242440-01	-0.109620-01	-0.125760-04
-0.227870-02	0.269910-03	0.123450-02	-0.459540-05	0.218800 00	0.115170 01	-0.453850 00	0.323160-02	-0.188560-02	0.115750-04
0.854700-02	-0.101240-02	-0.206900-02	0.837900-05	-0.543540 00	-0.453850 00	0.436390 00	-0.116950-01	0.679830-02	0.401000-05
0.218050-02	-0.313540-03	0.915210-03	-0.344520-05	0.242440-01	0.323160-02	-0.118950-01	0.281720 00	-0.119270 00	-0.356990-02
-0.150630-02	0.178410-03	-0.337240-03	0.136610-05	-0.109620-01	-0.188560-02	0.679830-02	-0.119270 00	0.655970-01	0.799920-03
-0.282900-05	0.335090-06	-0.445450-04	0.180400-06	-0.125760-04	0.115750-04	0.401000-05	-0.356990-02	0.799920-03	0.426280-03
0.123610-01	-0.128540-02	-0.243480-01	0.288490-04	0.286850 00	0.253770-01	-0.149180 00	0.905340-01	-0.419370-01	-0.534720-03
-0.161530-02	0.191320-03	0.183350-01	-0.742530-04	-0.562470-01	0.644940-01	0.143020-01	-0.227510-01	0.124900-01	0.155490-03
-0.131730 00	0.156040-01	-0.467670-01	0.189400-03	-0.268290 01	-0.171230 01	0.200840 01	-0.682010 00	0.377130 00	0.397360-02
-0.344770-04	0.406370-05	-0.775840-05	0.314200-07	0.454780-04	-0.615420-03	0.143640-03	-0.692930-03	0.131570-03	0.785670-04

-0.206270-01	0.921760-02	0.944300-01	-0.132450-04	
0.122670-02	-0.612330-03	-0.627310-02	0.300760-04	
-0.132060-01	0.198440-01	-0.210670 00	0.343240-05	
0.151060-02	-0.226470-02	0.240430-01	-0.504670-04	
-0.244450-01	-0.941750-02	-0.145020 00	0.594860-04	
-0.156770-01	0.173330-01	-0.127770 00	-0.357860-05	
-0.154350-02	-0.272240-02	0.881830-01	-0.422500-03	
0.809730 00	-0.533810 00	0.454310 00	0.726050-03	
-0.491000 00	0.406600 00	-0.101580 01	-0.116940-02	
-0.652840-01	-0.376250-01	0.192020 01		
0.123610-01	-0.161530-02	-0.131730 00	-0.344770-04	
-0.128540-02	0.191320-03	0.156040-01	0.406370-05	
-0.243480-01	0.183350-01	-0.467670-01	-0.775840-05	
0.288490-04	-0.742530-04	0.189400-03	0.314200-07	
0.286850 00	-0.562470-01	-0.268290 01	0.454780-04	
0.253770-01	0.644940-01	-0.171230 01	-0.615420-03	
-0.149180 00	0.143020-01	0.200840 01	0.143640-03	
0.905340-01	-0.227510-01	-0.682010 00	-0.692930-03	
-0.419370-01	0.124900-01	0.377130 00	0.131570-03	
-0.534720-03	0.155490-03	0.397360-02	0.785670-04	
0.856370 00	-0.270530 00	-0.609480 01	-0.391190-02	
-0.270530 00	0.284580 00	-0.175310 01	-0.548380-03	
-0.609480 01	-0.175310 01	0.128420 03	0.391940-01	
-0.391190-02	-0.548380-03	0.391940-01	0.363080-03	

COMPUTATIONAL DETERMINANT CHECK= 0.24197790 41

JOINT LEAST SQUARES ESTIMATES

SELECTION NO.	COEFICIENTS			
	CONSTANT	B1	B2	B3
1	C.468767	0.103172		
2	38.494036	-0.540984		
3	C.C74922	0.524027	0.246046	
4	-C.960958	3.851363	-1.405892	
5	-C.209615	0.254045		
6	C.621279	0.C03113		
7	-1.392054	7.731775	3.110125	
8	-C.118280	0.3C6717	0.091989	
9	3.220697	3.745272	-57.541979	C.C51347

TABLE OF PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	46.961441	35.272428	27.270000	23.583414
2	42.750668	31.168716	28.390000	19.586765
3	42.922090	31.339508	28.400000	19.756926
4	53.073715	41.566213	51.610000	30.058711
5	49.861508	38.425092	40.040000	26.988675
6	48.972870	37.563356	39.150000	26.153842
7	48.323039	36.920174	41.020000	25.517308
8	48.972870	37.563356	38.200000	26.153842
9	50.188199	38.747780	34.570000	27.309360
10	48.972870	37.563356	35.910000	26.153842
11	48.972870	37.563356	50.980000	26.153842
12	48.972870	37.563356	37.340000	26.153842
13	50.063905	38.632108	37.320000	27.200311
14	49.988179	38.552669	33.690000	27.117159
15	24.232777	12.503649	11.320000	0.774521
16	22.451310	10.721117	11.510000	-1.009075
17	22.451310	10.721117	12.080000	-1.009075
18	22.451310	10.721117	12.490000	-1.009075
19	22.894977	11.058243	8.700000	-0.778491
20	20.939221	9.094539	12.930000	-2.750144
21	29.266917	17.731768	18.490000	6.196619
22	38.334671	26.840069	25.090000	15.345467
23	34.482818	23.049254	23.990000	11.615689
24	34.362989	22.933478	44.700000	11.503967
25	35.338190	23.832306	20.250000	12.326423
26	46.757843	35.181267	38.490000	23.604691
27	43.244188	31.718632	27.270000	20.193076

MODEL C

JOINT GENERALIZED LEAST SQUARES

PREDICTION INTERVAL VALUES

LEAST SQUARES COVARIANCE MATRIX

0.483100D-01	-0.285090D-02	0.124660D-01	-0.172170D-02	0.980830D-03	-0.509770D-03	-0.116020D-03	0.416800D-03	-0.240350D-03	-0.673540D-04
-0.285090D-02	0.189380D-03	-0.100220D-02	0.114370D-03	-0.594070D-04	0.338640D-04	0.770740D-05	-0.238280D-04	0.159670D-04	0.447430D-05
0.124660D-01	-0.100220D-02	0.770620D 01	-0.760940D 00	0.104010D-02	0.100870D-02	0.551640D-04	-0.222400D-01	0.197320D-01	0.422770D-02
-0.172170D-02	0.114370D-03	-0.760940D 00	0.868410D-01	0.139290D-03	-0.115120D-03	-0.629550D-05	0.328910D-02	-0.225190D-02	-0.482480D-03
0.980830D-03	-0.594070D-04	-0.104010D-02	0.139290D-03	0.249760D-01	-0.123240D-01	-0.337270D-02	-0.955310D-02	0.561340D-02	0.168040D-02
-0.509770D-03	0.338640D-04	0.100870D-02	-0.115120D-03	-0.123240D-01	0.277070D-01	-0.521870D-02	0.104690D-01	-0.895300D-02	0.349710D-02
-0.116020D-03	0.770740D-05	0.551640D-04	-0.629550D-05	-0.337270D-02	-0.521870D-02	0.279170D-02	-0.773710D-03	0.116820D-02	-0.168660D-02
0.416800D-03	-0.238280D-04	-0.222400D-01	0.328910D-02	-0.955310D-02	0.104690D-01	-0.773710D-03	0.424310D 01	-0.315030D 01	0.108340D 01
-0.240350D-03	0.159670D-04	0.197320D-01	-0.225190D-02	0.561340D-02	-0.895300D-02	0.116820D-02	-0.315030D 01	0.300650D 01	-0.193290D 01
-0.673540D-04	0.447430D-05	0.422770D-02	-0.482480D-03	0.168040D-02	0.349710D-02	-0.168660D-02	0.108340D 01	-0.193290D 01	0.319900D 01
0.220340D-01	-0.128690D-02	0.210580D 00	-0.298000D-01	0.131060D 00	-0.559220D-01	-0.192720D-01	-0.472510D-02	0.256630D-02	0.100850D-02
-0.229460D-02	0.152430D-03	-0.309290D-01	0.352980D-02	-0.140600D-01	0.662380D-02	0.228270D-02	0.470120D-03	-0.303970D-03	-0.119460D-03
-0.154040D-01	0.927480D-03	0.852570D-01	-0.113180D-01	0.285000D-02	-0.253450D-02	0.261080D-05	-0.175650D 00	0.118650D 00	-0.365750D-02
0.565420D-04	-0.375610D-05	-0.401630D-03	0.458360D-04	-0.106190D-04	0.102640D-04	-0.105730D-07	0.645040D-03	-0.480510D-03	0.149740D-04
-0.127490D-01	0.712870D-03	0.193500D-01	-0.288550D-02	-0.329640D-01	0.961740D-02	0.598810D-02	0.884440D-01	-0.481090D-01	-0.710220D-02
0.822020D-03	-0.546070D-04	0.233080D-02	-0.266000D-03	0.352960D-03	-0.279260D-02	0.808470D-03	0.113760D-01	-0.471510D-01	0.109300D 00
0.591840D-02	-0.393160D-03	-0.151210D-01	0.172570D-02	0.164130D-01	-0.473740D-02	-0.365190D-02	-0.424900D-01	0.405500D-01	-0.260700D-01
0.949110D-02	-0.533660D-03	-0.107590D 00	0.161510D-01	0.247770D-02	-0.898740D-03	-0.396060D-03	0.467280D-01	-0.232920D-01	-0.107600D-01
-0.453530D-02	0.301280D-03	0.813040D-01	-0.927870D-02	-0.121310D-02	0.452160D-03	0.236290D-03	-0.198910D-01	0.121400D-01	0.708620D-02
-0.192560D-04	0.127920D-05	-0.776770D-04	0.886470D-05	-0.103640D-04	0.185140D-04	-0.281120D-05	-0.349660D-03	0.354740D-03	-0.273870D-03
-0.215900D-01	0.129260D-02	-0.198210D-03	0.259170D-04	0.257930D-01	-0.121630D-01	-0.353070D-02	0.841980D 00	-0.490420D 00	-0.133870D 00
0.797390D-02	-0.529710D-03	0.575180D-04	-0.656410D-05	-0.911300D-02	0.146970D-01	-0.195000D-02	-0.593330D 00	0.460170D 00	-0.650090D-01
0.134720 00	-0.894950D-02	0.232660D-02	-0.265520D-03	-0.173960D 00	-0.125820 00	0.967350D-01	0.125780 01	-0.224400D 01	0.371390 01
-0.216760D-05	0.144000D-06	-0.546040D-07	0.623160D-08	-0.341730D-04	0.492040D-04	-0.536510D-05	-0.934980D-03	0.166800D-02	-0.276070D-02

0.220340D-01	-0.229460D-02	-0.154040D-01	0.565420D-04	-0.127490D-01	0.822020D-03	0.591840D-02	0.949110D-02	-0.453530D-02	-0.192560D-04
-0.128690D-02	0.152430D-03	0.927480D-03	-0.375610D-05	0.712870D-03	-0.546070D-04	-0.393160D-03	-0.533660D-03	0.301280D-03	0.127920D-05
0.210580D 00	-0.309290D-01	0.852570D-01	-0.401630D-03	0.193500D-01	0.233080D-02	-0.151210D-01	-0.107590D 00	0.813040D-01	-0.776770D-04
-0.298000D-01	0.352980D-02	-0.113180D-01	0.458360D-04	-0.288550D-02	-0.266000D-03	0.172570D-02	0.161510D-01	-0.927870D-02	0.886470D-05
0.131060D 00	-0.140600D-01	0.285000D-02	-0.106190D-04	-0.329640D-01	0.352960D-03	0.164130D-01	0.247770D-02	-0.121310D-02	-0.103640D-04
-0.559220D-01	0.662380D-02	-0.253450D-02	0.102640D-04	0.961740D-02	-0.279260D-02	-0.473740D-02	-0.898740D-03	0.452160D-03	0.185140D-04
-0.192720D-01	0.228270D-02	0.261080D-05	-0.105730D-07	0.598810D-02	0.808470D-03	-0.365190D-02	-0.396060D-03	0.236290D-03	-0.281120D-05
-0.472510D-02	0.470120D-03	-0.175650D 00	0.645040D-03	0.884440D-01	0.113760D-01	-0.424900D-01	0.467280D-01	-0.198910D-01	-0.349660D-03
0.256630D-02	-0.303970D-03	0.118650D 00	-0.480510D-03	-0.481090D-01	-0.471510D-01	0.405500D-01	-0.232920D-01	0.121400D-01	0.354740D-03
0.100850D-02	-0.119460D-03	-0.369750D-02	0.149740D-04	-0.710220D-02	0.109300D 00	-0.260700D-01	-0.107600D-01	0.708620D-02	-0.273870D-03
0.943970D-01	-0.973950D-02	-0.992360D-02	0.358660D-04	-0.175550D-01	0.822240D-03	0.814870D-02	-0.285320D-02	0.135440D-02	0.365280D-05
-0.973950D-02	0.115360D-02	0.104900D-02	-0.424820D-05	0.173240D-02	-0.973920D-04	-0.964190D-03	0.282690D-03	-0.160420D-03	-0.433660D-06
-0.992360D-02	0.104900D-02	0.358570D-01	-0.136310D-03	0.271950D-02	-0.344650D-03	-0.123260D-02	0.492030D-03	-0.234240D-03	-0.262410D-05
0.358660D-04	-0.424820D-05	-0.136310D-03	0.552030D-06	-0.938920D-05	0.139580D-05	0.499180D-05	-0.171920D-05	0.948610D-06	0.106270D-07
-0.175550D-01	0.173240D-02	0.271950D-02	-0.938520D-05	0.106930D 01	0.259090D 00	-0.565440D 00	0.249740D-01	-0.113980D-01	0.725930D-05
0.822240D-03	-0.973920D-04	-0.344650D-03	0.139580D-05	0.259090D 00	0.147190D 01	-0.554180D 00	0.377250D-03	-0.768100D-04	-0.412400D-04
0.814870D-02	-0.964190D-03	-0.123260D-02	0.499180D-05	-0.565440 00	-0.554180D 00	0.476590D 00	-0.115270D-01	0.654820D-02	0.155280D-04
-0.285320D-02	0.282690D-03	0.492030D-03	-0.171920D-05	0.249740D-01	0.377250D-03	-0.115270D-01	0.288280D 00	-0.121990D 00	-0.421670D-02
0.135440D-02	-0.160420D-03	-0.234240D-03	0.948610D-06	-0.113980D-01	-0.768100D-04	0.654820D-02	-0.121990D 00	0.669560D-01	0.858540D-03
0.365280D-05	-0.432660D-06	-0.262410D-05	0.106270D-07	-0.725930D-05	-0.412400D-04	0.155280D-04	-0.421670D-02	0.858540D-03	0.460960D-02
0.141560D-01	-0.149800D-02	-0.296650D-01	0.109980D-03	0.296760D 00	0.250740D-01	-0.154770D 00	0.103960D 00	-0.482940D-01	-0.126090D-02
-0.466650D-02	0.545620D-03	0.166260D-01	-0.673330D-04	-0.518480D-01	0.663500D-01	0.113970D-01	-0.214660D-01	0.117820D-01	0.151080D-03
-0.100030D 00	0.118480D-01	0.695430D-01	-0.281630D-03	-0.292130 01	-0.171040 01	0.214440 01	-0.885390 00	0.485970 00	0.623130D-02
0.449800D-05	-0.532780D-06	0.212810D-04	-0.861840D-07	-0.519370D-04	-0.907810D-03	0.280040D-03	-0.983530D-03	0.187670D-03	0.111240D-03

-0.215900D-01	0.797390D-02	0.134720 00	-0.216760D-05	
0.129260D-02	-0.529710D-03	-0.894950D-02	0.144000D-06	
-0.198210D-03	0.575180D-04	0.232660D-02	-0.546040D-07	
0.259170D-04	-0.656410D-05	-0.265520D-03	0.623160D-08	
0.257930D-01	-0.911300D-02	-0.173960D 00	-0.341730D-04	
-0.121630D-01	0.146970D-01	-0.125820 00	0.492040D-04	
-0.353070D-02	-0.195000D-02	0.967350D-01	-0.536510D-05	
0.841980D 00	-0.593330 00	0.125780 01	-0.934980D-03	
-0.490430 00	0.460170D 00	-0.224400D 01	0.166800D-02	
-0.133870D 00	-0.650090D-01	0.371390 01	-0.276070D-02	
0.141560D-01	-0.466650D-02	-0.100030D 00	0.449800D-05	
-0.149800D-02	0.545620D-03	0.118480D-01	-0.532780D-06	
-0.296650D-01	0.166260D-01	0.695430D-01	0.212810D-04	
0.109980D-03	-0.673330D-04	-0.281630D-03	-0.861840D-07	
0.296760D 00	-0.518480D-01	-0.292130 01	-0.519370D-04	
0.250740D-01	0.663500D-01	-0.171040 01	-0.907810D-03	
-0.154770 00	0.113970D-01	0.214440 01	0.280040D-03	
0.103960 00	-0.214660D-01	-0.885390 00	-0.983530D-03	
-0.482940D-01	0.117820D-01	0.485970 00	0.187670D-03	
-0.126090D-02	0.151080D-03	0.623130D-02	0.111240D-03	
0.105090D 01	-0.227430 00	-0.965150 01	-0.776950D-02	
-0.227430 00	0.356210 00	-0.387590 01	-0.124020D-02	
-0.965150 01	-0.387590 01	0.224840 03	0.815090D-01	
-0.776950D-02	-0.124020D-02	0.815090D-01	0.716000D-03	

TABLE OF LS CORRELATED PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	46.179220	34.234324	27.270000	22.289429
2	42.205896	30.422669	28.390000	18.639443
3	42.380366	30.596566	28.400000	18.812766
4	52.642602	41.020917	51.610000	29.399232
5	49.639078	38.147348	40.040000	26.655619
6	48.794931	37.317595	39.150000	25.840259
7	48.169940	36.699251	41.020000	25.228561
8	48.794931	37.317595	38.200000	25.840259
9	49.955463	38.462943	34.570000	26.970424
10	48.794931	37.317595	35.910000	25.840259
11	48.794931	37.317595	50.980000	25.840259
12	48.794931	37.317595	37.340000	25.840259
13	49.868924	38.382012	37.320000	26.895100
14	49.789118	38.300706	33.690000	26.812293
15	24.544679	12.757103	11.320000	0.969526
16	22.891935	11.106287	11.510000	-0.679361
17	22.891935	11.106287	12.080000	-0.679361
18	22.891935	11.106287	12.490000	-0.679361
19	23.634649	11.745668	8.700000	-0.143313
20	21.821871	9.921718	12.930000	-1.978434
21	29.830509	18.248583	18.490000	6.666657
22	38.584367	26.911125	25.090000	15.237882
23	34.969159	23.383477	23.990000	11.797795
24	34.821636	23.241512	44.700000	11.661388
25	35.865046	24.165638	20.250000	12.466230
26	47.221375	35.535042	38.490000	23.848708
27	43.930199	32.312108	27.270000	20.694018

TABLE OF LS (INDEPEND) PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	45.244064	34.234222	27.269989	23.224365
2	41.287399	30.422546	28.389999	19.557678
3	41.473557	30.596436	28.399994	19.719299
4	51.648346	41.020844	51.609985	30.393326
5	48.606689	38.147217	40.039993	27.687729
6	47.776276	37.317474	39.149994	26.858658
7	47.151108	36.699127	41.019989	26.247131
8	47.776276	37.317474	38.199997	26.858658
9	48.924454	38.462814	34.569992	28.001160
10	47.776276	37.317474	35.909988	26.858658
11	47.776276	37.317474	50.979996	26.858658
12	47.776276	37.317474	37.339996	26.858658
13	48.839310	38.381882	37.319992	27.924438
14	48.758224	38.300552	33.689987	27.842865
15	23.405258	12.757086	11.320000	2.108913
16	21.751877	11.106257	11.509999	0.460636
17	21.751877	11.106257	12.080000	0.460636
18	21.751877	11.106257	12.490000	0.460636
19	22.433395	11.745620	8.700000	1.057842
20	20.619278	9.921659	12.929999	-0.775964
21	28.684082	18.248489	18.489990	7.812886
22	37.507278	26.911041	25.089996	16.314789
23	33.909637	23.383377	23.989990	12.857104
24	33.762192	23.241409	44.699997	12.720620
25	34.811874	24.165512	20.250000	13.519141
26	46.174057	35.534973	38.489990	24.895874
27	42.866684	32.312012	27.269989	21.757324

MODEL D - 4 CERs - 36 OBSERVATIONS

```
MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 1

SELECTION CARD 037 121
        LEAST SQUARES REGRESSION COEFICIENTS

    0.470823     0.001158
STD. ERROR OF ESTIMATE=         0.804216

        COVARIANCE MATRIX OF THE ESTIMATES

    0.13244D 00 -0.3583CD-04
   -0.3583CD-04   C.11214D-07
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.772147


MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 2

SELECTION CARD 038 23341
        LEAST SQUARES REGRESSION COEFICIENTS

    0.005228     0.286414    -0.651530
STD. ERROR OF ESTIMATE=         1.070823

        COVARIANCE MATRIX OF THE ESTIMATES

    0.26784D 00 -C.21842D C2  0.36699D 02
   -0.21842D 02  C.36286D C4 -0.21771D 05
    0.36699D 02 -0.21771D C5  0.21581D 06
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.891135
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MULTIPLE REGRESSION.....GENRAL
     SELECTION..... 3

SELECTION CARD 039 22128
        LEAST SQUARES REGRESSION COEFICIENTS

   -6.785734     0.0C585C     6.664107
STD. ERRCR OF ESTIMATE=         4.824125

        COVARIANCE MATRIX CF THE ESTIMATES

    0.59974D 01 -C.21718D-C2  0.21987D 01
   -0.21718D-02  0.10359D-C5 -0.15753D-02
    0.21987D 01 -0.15753D-C2  0.39246D 01
ADJUSTED MULTIPLE CORRELATICN COEFICIENT 0.844043


MULTIPLE REGRESSICN.....GENRAL
     SELECTION..... 4

SELECTION CARD 04C 21423
        LEAST SQUARES REGRESSION COEFICIENTS

   -1.014043     0.005553     0.002013
STD. ERRCR OF ESTIMATE=         2.233319

        COVARIANCE MATRIX CF THE ESTIMATES

    0.8557CD CO -C.54173D-C3 -0.19542D-05
   -0.54173D-03  C.43047D-C6 -0.36635D-07
   -0.19542D-05 -C.36635D-C7  0.68360D-07
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.824401
```

MATRIX OF RESIDUALS FOR SELECTIONS

CASE NO.	SELECTION NUMBER 1	2	3	4
1	0.14339	-0.00057	-2.94288	-5.43512
2	0.20339	0.00005	-2.58288	0.59742
3	0.48339	0.00041	-2.97288	-0.33258
4	-0.12363	-0.00005	0.31065	3.61752
5	-0.57775	-0.00155	0.49625	1.27806
6	-0.57848	-0.00041	0.19305	-1.34539
7	-1.41931	0.00099	1.17350	0.52246
8	-0.58848	0.00030	-2.30695	0.63461
9	0.26940	-0.00048	-4.10020	-2.24420
10	-0.19848	0.00064	-2.75695	-1.37539
11	-0.15848	-0.00050	10.26305	1.12461
12	0.23152	0.00045	-3.78695	0.34461
13	0.43655	-0.00109	-1.64665	-0.74307
14	-0.36260	-0.00116	-3.56135	-0.63637
15	0.83358	-0.00115	1.19070	-3.01340
16	0.39358	0.00114	0.78070	-0.51681
17	0.64358	-0.00048	1.07070	0.34319
18	0.87358	0.00151	1.11070	0.25319
19	-0.31749	-0.00056	-0.44880	-2.12091
20	0.12251	0.00116	1.12120	2.08657
21	-0.23298	-0.00123	0.72584	-0.66180
22	-0.42351	-0.00064	-3.30648	0.47806
23	-0.68065	-0.00066	-2.82003	3.01800
24	-0.87518	-0.00025	15.13867	5.20251
25	-0.53170	-0.00037	-3.49378	-0.29083
26	-0.48941	-0.00036	2.43806	-1.28208
27	-0.47153	-0.00033	-4.35519	-0.08743
28	1.95286	0.00212	5.04403	4.84444
29	-0.15714	0.00077	-4.77597	0.14444
30	0.69801	0.00134	10.34475	1.63506
31	0.37146	-0.00015	5.04655	1.43833
32	1.93801	0.00042	-7.40525	-3.41301
33	1.56801	0.00130	-5.41525	-1.06301
34	-0.75720	-0.00024	4.30830	1.10806
35	-1.30152	-0.00069	-1.21455	-2.53988
36	-0.91731	0.00029	-0.86380	-1.56988

MATRIX OF CORRELATIONS BETWEEN SELECTIONS

SELECTION NO.	1	2	3	4
1	1.000000	0.441009	-0.106147	-0.054891
2	0.441009	1.000000	0.084968	0.278739
3	-0.106147	0.084968	1.000000	0.555159
4	-0.054891	0.278739	0.555159	1.000000

JOINT GENERALIZED LEAST SQUARES REGRESSION OUTPUT

COVARIANCE MATRIX OF THE JOINT ESTIMATES

0.11967D 00	-0.32144D-04	-0.32757D-04	0.18977D-02	0.15322D-01	-0.85815D-01
-0.32144D-04	0.10061D-07	0.12835D-07	-0.59397D-06	-0.47957D-05	0.23528D-04
-0.32757D-04	0.12835D-07	0.15394D-06	-0.11363D-04	0.95645D-05	-0.32109D-04
0.18977D-02	-0.59397D-06	-0.11363D-04	0.17446D-02	-0.96895D-02	0.35733D-02
0.15322D-01	-0.47957D-05	0.95645D-05	-0.96895D-02	0.10216D 00	-0.33900D-02
-0.85815D-01	0.23528D-04	-0.32109D-04	0.35733D-02	-0.33900D-02	0.50814D 01
0.23743D-04	-0.74313D-08	0.13648D-07	-0.11461D-05	0.78328D-06	-0.17264D-02
-0.95107D-03	0.29768D-06	-0.29105D-05	0.12235D-03	0.12288D-02	0.14220D 01
-0.26754D-01	0.75761D-05	-0.46303D-04	0.41524D-02	0.72026D-02	0.98139D 00
0.18501D-04	-0.57906D-08	0.45298D-07	-0.29554D-05	-0.71024D-05	-0.62762D-03
-0.32204D-06	0.10079D-09	0.97977D-09	-0.33628D-06	0.29607D-05	0.13747D-06

0.23743D-04	-0.95107D-03	-0.26754D-01	0.18501D-04	-0.32204D-06	
-0.74313D-08	0.29768D-06	0.75761D-05	-0.57906D-08	0.10079D-09	
0.13648D-07	-0.29105D-05	-0.46303D-04	0.45298D-07	0.97977D-09	
-0.11461D-05	0.12235D-03	0.41524D-02	-0.29554D-05	-0.33628D-06	
0.78328D-06	0.12288D-02	0.72026D-02	-0.71024D-05	0.29607D-05	
-0.17264D-02	0.14220D 01	0.98139D 00	-0.62762D-03	0.13747D-06	
0.76717D-06	-0.10035D-02	-0.26667D-03	0.19983D-06	0.36243D-08	
-0.10035D-02	0.24703D 01	0.31761D-01	-0.14995D-04	-0.16224D-04	
-0.26667D-03	0.31761D-01	0.74682D 00	-0.46959D-03	0.78863D-06	
0.19983D-06	-0.14995D-04	-0.46959D-03	0.36801D-06	-0.22551D-07	
0.36243D-08	-0.16224D-04	0.78863D-06	-0.22551D-07	0.39376D-07	

COMPUTATIONAL DETERMINANT CHECK= 0.1366244D 44

JOINT LEAST SQUARES ESTIMATES

SELECTION NO.	COEFFICIENTS			
	CONSTANT	B1	B2	B3
1	0.597865	0.001118		
2	0.005293	0.274581	-0.574847	
3	-6.307849	0.005587	7.167954	
4	-0.680097	0.005401	0.001832	

TABLE OF PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	46.779931	33.966494	27.270000	21.153057
2	41.496805	28.721677	28.390000	15.946549
3	41.567747	28.791345	28.400000	16.014943
4	57.000953	44.309232	51.610000	31.617510
5	52.267304	39.608704	40.040000	26.950104
6	51.668665	39.009967	39.150000	26.351269
7	50.944936	38.280008	41.020000	25.615080
8	51.668665	39.009967	38.200000	26.351269
9	52.574953	39.918795	34.570000	27.262637
10	51.668665	39.009967	35.910000	26.351269
11	51.668665	39.009967	50.980000	26.351269
12	51.668665	39.009967	37.340000	26.351269
13	52.836926	40.183513	37.320000	27.530101
14	52.655075	40.000924	33.690000	27.346774
15	25.302176	12.369640	11.320000	-0.562896
16	22.971987	10.045793	11.510000	-2.880401
17	22.971987	10.045793	12.080000	-2.880401
18	22.971987	10.045793	12.490000	-2.880401
19	24.486919	11.580702	8.700000	-1.325514
20	21.968723	9.070958	12.930000	-3.826807
21	31.529251	18.733967	18.490000	5.938682
22	42.126864	29.364184	25.090000	16.601505
23	36.868823	24.138036	23.990000	11.407248
24	36.853816	24.125882	44.700000	11.397948
25	36.939294	24.211309	20.250000	11.483325
26	49.691832	36.953751	38.490000	24.215669
27	45.270486	32.558521	27.270000	19.846556
28	60.623622	47.915886	67.590000	35.208151
29	60.631846	47.924124	46.230000	35.216402
30	87.494562	74.356561	93.900000	61.218559
31	79.445994	66.331784	75.310000	53.217575
32	78.264738	65.180421	59.300000	52.096105
33	78.264738	65.180421	62.900000	52.096105
34	87.973501	74.452855	80.040000	60.932210
35	77.611894	64.175077	59.500000	50.738260
36	77.615738	64.176842	62.170000	50.737946

MODEL D

JOINT GENERALIZED LEAST SQUARES

PREDICTION INTERVAL VALUES

LEAST SQUARES COVARIANCE MATRIX

0.12508D 00	-0.33839D-04	-0.36173D-04	0.21077D-02	0.16007D-01	-0.78464D-01
-0.33839D-04	0.10591D-07	0.13904D-07	-0.65968D-06	-0.50100D-05	0.21227D-04
-0.36173D-04	0.13904D-07	0.23358D-06	-0.19049D-04	0.32005D-04	-0.42120D-04
0.21077D-02	-0.65968D-06	-0.19049D-04	0.31645D-02	-0.18987D-01	0.27176D-02
0.16007D-01	-0.50100D-05	0.32005D-04	-0.18987D-01	0.18821D 00	0.15430D-01
-0.78464D-01	0.21227D-04	-0.42120D-04	0.27176D-02	0.15430D-01	0.54976D 01
0.21227D-04	-0.66438D-08	0.16507D-07	-0.97927D-06	-0.37052D-05	-0.19908D-02
0.98412D-16	-0.31542D-19	-0.17017D-05	0.56932D-03	-0.49736D-02	0.20155D 01
-0.16762D-01	0.44489D-05	-0.54705D-04	0.44311D-02	0.11613D-01	0.10210D 01
0.10857D-04	-0.33981D-08	0.51459D-07	-0.30981D-05	-0.11034D-04	-0.65541D-03
-0.17567D-06	0.54984D-10	0.13540D-08	-0.45879D-06	0.40236D-05	-0.38304D-05

0.21227D-04	0.53176D-16	-0.16762D-01	0.10857D-04	-0.17567D-06
-0.66438D-08	-0.15579D-19	0.44489D-05	-0.33981D-08	0.54984D-10
0.16507D-07	-0.17017D-05	-0.54705D-04	0.51459D-07	0.13540D-08
-0.97927D-06	0.56932D-03	0.44311D-02	-0.30981D-05	-0.45879D-06
-0.37052D-05	-0.49736D-02	0.11613D-01	-0.11034D-04	0.40236D-05
-0.19908D-02	0.20155D 01	0.10210D 01	-0.65541D-03	-0.38304D-05
0.94953D-06	-0.14440D-02	-0.27963D-03	0.20776D-06	0.69809D-08
-0.14440D-02	0.35975D 01	0.34172D-01	-0.11604D-04	-0.25579D-04
-0.27963D-03	0.34172D-01	0.78440D 00	-0.49659D-03	-0.17913D-05
0.20776D-06	-0.11604D-04	-0.49659D-03	0.39459D-06	-0.33582D-07
0.69809D-08	-0.25579D-04	-0.17913D-05	-0.33582D-07	0.62664D-07

TABLE OF LS CORRELATED PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	47.496196	34.597153	27.270000	21.698110
2	41.673262	28.833934	28.390000	15.994607
3	41.753755	28.912179	28.400000	16.070603
4	57.334560	44.605451	51.610000	31.876342
5	52.027124	39.350252	40.040000	26.673380
6	51.422412	38.745332	39.150000	26.068252
7	50.687839	38.001443	41.020000	25.315047
8	51.422412	38.745332	38.200000	26.068252
9	52.340885	39.667734	34.570000	26.994582
10	51.422412	38.745332	35.910000	26.068252
11	51.422412	38.745332	50.980000	26.068252
12	51.422412	38.745332	37.340000	26.068252
13	52.614181	39.944989	37.320000	27.275797
14	52.431483	39.760984	33.690000	27.090484
15	25.116778	12.164660	11.320000	-0.787458
16	22.551253	9.607907	11.510000	-3.335440
17	22.551253	9.607907	12.080000	-3.335440
18	22.551253	9.607907	12.490000	-3.335440
19	24.283593	11.355201	8.700000	-1.573191
20	21.514075	8.597391	12.930000	-4.319294
21	31.594248	18.775287	18.490000	5.956326
22	42.729326	29.902177	25.090000	17.075028
23	36.954224	24.177540	23.990000	11.400856
24	36.935584	24.163164	44.700000	11.390745
25	37.018018	24.245725	20.250000	11.473432
26	49.791322	37.002960	38.490000	24.214598
27	44.883587	32.133265	27.270000	19.382943
28	60.699060	47.980272	67.590000	35.261484
29	60.707653	47.988864	46.230000	35.270076
30	88.383230	75.122716	93.900000	61.862201
31	79.428899	66.233432	75.310000	53.037964
32	78.209221	65.039599	59.300000	51.869977
33	78.209221	65.039599	62.900000	51.869977
34	89.737422	76.038214	80.040000	62.339005
35	78.311529	64.757111	59.500000	51.202693
36	78.317571	64.760783	62.170000	51.203995

TABLE OF LS (INDEPEND) PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	45.842148	34.597580	27.269989	23.352997
2	40.020111	28.835068	28.389999	17.650009
3	40.093536	28.913330	28.399994	17.733109
4	55.642090	44.605728	51.609985	33.569351
5	50.345749	39.351334	40.039993	28.356903
6	49.722260	38.746384	39.149994	27.770493
7	48.965347	38.002487	41.019989	27.039612
8	49.722260	38.746384	38.199997	27.770493
9	50.668243	39.668839	34.569992	28.669418
10	49.722260	38.746384	35.909988	27.770493
11	49.722260	38.746384	50.979996	27.770493
12	49.722260	38.746384	37.339996	27.770493
13	50.939285	39.946075	37.319992	28.952850
14	50.747025	39.762070	33.689987	28.777100
15	23.195160	12.164678	11.320000	1.134190
16	20.629440	9.608246	11.509999	-1.412958
17	20.629440	9.608246	12.080000	-1.412958
18	20.629440	9.608246	12.490000	-1.412958
19	22.365707	11.355088	8.700000	0.344466
20	19.597290	8.597610	12.929999	-2.402081
21	29.731567	18.775330	18.489990	7.819076
22	40.925278	29.902405	25.089996	18.879517
23	35.152039	24.178421	23.989990	13.204800
24	35.132599	24.164032	44.699997	13.195461
25	35.219788	24.246597	20.250000	13.273404
26	48.000046	37.003036	38.489990	26.006012
27	43.078568	32.134003	27.269989	21.189423
28	59.034042	47.981674	67.589996	36.929291
29	59.042648	47.990265	46.229996	36.937866
30	86.897797	75.123077	93.899994	63.348343
31	77.875992	66.235046	75.309998	54.594086
32	76.669571	65.041183	59.299988	53.412781
33	76.669571	65.041183	62.899994	53.412781
34	88.234818	76.038818	80.039993	63.842804
35	76.781921	64.759079	59.500000	52.736221
36	76.785767	64.762741	62.169998	52.739700

MODEL E - 4 CERs - 27 OBSERVATIONS

```
MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 1

SELECTION CARD 037 121
        LEAST SQUARES REGRESSION COEFICIENTS

    1.207170     0.000825
STD. ERROR OF ESTIMATE=        0.495120

        COVARIANCE MATRIX OF THE ESTIMATES

    0.119660 00 -0.427930-04
   -0.427930-04  0.165600-07
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.606372



MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 2

SELECTION CARD 038 133
        LEAST SQUARES REGRESSION COEFICIENTS

    0.662174    -0.755852
STD. ERROR OF ESTIMATE=        0.912000

        COVARIANCE MATRIX OF THE ESTIMATES

    0.228570 00 -0.225700 01
   -0.225700 01  0.257570 02
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.894681
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MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 3

SELECTION CARD 039 22128
        LEAST SQUARES REGRESSION COEFICIENTS

   -4.188301     0.004682     7.148997
STD. ERROR OF ESTIMATE=        4.414830

        COVARIANCE MATRIX OF THE ESTIMATES

    0.13266D 02 -0.57465D-02  0.47872D 01
   -0.57465D-02  0.27811D-05 -0.29906D-02
    0.47872D 01 -0.29906D-02  0.61073D 01
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.687568


MULTIPLE REGRESSION.....ESCORT
     SELECTION..... 4

SELECTION CARD 040 21422
        LEAST SQUARES REGRESSION COEFICIENTS

   -2.627433     0.007357     0.001560
STD. ERROR OF ESTIMATE=        2.051149

        COVARIANCE MATRIX OF THE ESTIMATES

    0.26712D 01 -0.23852D-02 -0.97503D-04
   -0.23852D-02  0.23061D-05  0.14967D-07
   -0.97503D-04  0.14967D-07  0.13923D-06
ADJUSTED MULTIPLE CORRELATION COEFICIENT 0.593167
```

MATRIX OF RESIDUALS FOR SELECTIONS

CASE NO.	SELECTION NUMBER 1	2	3	4
1	0.36221	-0.01455	-2.19580	-4.25904
2	0.42221	0.03284	-1.83580	0.47754
3	0.70221	0.03799	-2.22580	-0.45246
4	0.19227	0.03229	0.91279	4.47984
5	-0.24050	-0.06026	1.17315	0.67120
6	-0.24390	0.01815	0.86060	-1.86567
7	-1.11041	0.10234	1.75111	-0.05373
8	-0.25390	0.07535	-1.63940	0.11433
9	0.61900	0.03679	-3.38008	-2.82580
10	0.13610	0.10298	-2.08940	-1.89567
11	0.17610	0.01108	10.93060	0.60433
12	0.56610	0.08756	-3.11940	-0.17567
13	0.79849	-0.01591	-0.88331	-1.33730
14	-0.00667	-0.02876	-2.81903	-1.25765
15	0.53094	-0.19377	0.11191	-1.85765
16	0.09094	-0.10238	-0.29809	0.05048
17	0.34094	-0.16973	-0.00809	0.91048
18	0.57094	-0.08759	0.03191	0.82048
19	-0.59677	0.00624	-1.44582	-0.97135
20	-0.15677	0.13932	0.12418	2.61599
21	-0.35512	-0.00047	0.27904	0.34926
22	-0.26608	-0.02409	-2.77434	1.10732
23	-0.53557	-0.02506	-2.33112	2.45983
24	-0.73743	0.00409	15.60188	4.59384
25	-0.39496	0.00603	-3.03407	-0.89409
26	-0.32164	0.00865	2.52152	-0.66404
27	-0.28874	0.02137	-4.21916	-0.74479

MATRIX OF CORRELATIONS BETWEEN SELECTIONS

SELECTION NO.	1	2	3	4
1	1.000000	-0.255531	-0.280213	-0.317896
2	-0.255531	1.000000	-0.043023	0.070142
3	-0.280213	-0.043023	1.000000	0.490192
4	-0.317896	0.070142	0.490192	1.000000

JCINT GENERALIZED LEAST SQUARES REGRESSION OUTPUT

COVARIANCE MATRIX OF THE JOINT ESTIMATES

```
 0.10560D 00 -0.37611D-04  0.17057D-02 -0.23309D-01 -0.24853D 00
-0.37611D-04  0.14555D-07 -0.79041D-06  0.90205D-05  0.88216D-04
 0.17057D-02 -0.79041D-06  0.14789D-02 -0.14520D-01  0.35658D-02
-0.23309D-01  0.90205D-05 -0.14520D-01  0.16571D 00 -0.46347D-01
-0.24853D 00  0.88216D-04  0.35658D-02 -0.46347D-01  0.10517D 02
 0.88042D-04 -0.34071D-07 -0.16387D-05  0.18701D-04 -0.44006D-02
 0.93678D-03 -0.36253D-06  0.35994D-03 -0.41077D-02  0.31079D 01
-0.13655D 00  0.48646D-04 -0.16152D-02  0.22715D-01  0.19153D 01
 0.12291D-03 -0.47567D-07  0.19748D-05 -0.22537D-04 -0.17163D-02
-0.16259D-05  0.62922D-09 -0.75664D-07  0.86350D-06 -0.10190D-05
```

COMPUTATIONAL DETERMINANT CHECK= 0.84180930 34

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 0.88042D-04  0.93678D-03 -0.13655D 00  0.12291D-03 -0.16259D-05
-0.34071D-07 -0.36253D-06  0.48646D-04 -0.47567D-07  0.62922D-09
-0.16387D-05  0.35994D-03 -0.16152D-02  0.19748D-05 -0.75664D-07
 0.18701D-04 -0.41077D-02  0.22715D-01 -0.22537D-04  0.86350D-06
-0.44006D-02  0.31079D 01  0.19153D 01 -0.17163D-02 -0.10190D-05
 0.20649D-05 -0.19425D-02 -0.68831D-03  0.66468D-06  0.57203D-08
-0.19425D-02  0.39702D 01  0.19585D-01 -0.26538D-05 -0.28584D-04
-0.68831D-03  0.19585D-01  0.22262D 01 -0.19933D-02 -0.56919D-04
 0.66468D-06 -0.26538D-05 -0.19933D-02  0.19321D-05  0.39063D-08
 0.57203D-08 -0.28584D-04 -0.56919D-04  0.39063D-08  0.89723D-07
```

JCINT LEAST SQUARES ESTIMATES

SELECTION NO.	COEFICIENTS			
	CONSTANT	B1	B2	B3
1	1.072839	0.000877		
2	0.678334	-0.940282		
3	-4.779052	0.004788	7.804857	
4	-2.050096	0.006788	0.001575	

TABLE OF PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	42.744578	31.861400	27.270000	20.978222
2	38.144302	27.352879	28.390000	16.561456
3	38.566898	27.773446	28.400000	16.979995
4	53.430695	42.696682	51.610000	31.962670
5	48.907651	38.248580	40.040000	27.589510
6	48.522766	37.877430	39.150000	27.232095
7	48.297413	37.651677	41.020000	27.005941
8	48.522766	37.877430	38.200000	27.232095
9	49.023746	38.363142	34.570000	27.702538
10	48.522766	37.877430	35.910000	27.232095
11	48.522766	37.877430	50.980000	27.232095
12	48.522766	37.877430	37.340000	27.232095
13	49.289304	38.620256	37.320000	27.951207
14	49.290216	38.620112	33.690000	27.949909
15	24.638061	13.611625	11.320000	2.585188
16	22.729269	11.706895	11.510000	0.684520
17	22.729269	11.706895	12.080000	0.684520
18	22.729269	11.706895	12.490000	0.684520
19	21.843423	10.869270	8.700000	-0.104882
20	19.679577	8.711857	12.930000	-2.255862
21	27.853401	17.156160	18.490000	6.458919
22	38.739543	27.946116	25.090000	17.152688
23	34.119218	23.422719	23.990000	12.726120
24	34.179619	23.488163	44.700000	12.796707
25	34.140610	23.450805	20.250000	12.760999
26	46.752112	36.022949	38.490000	25.293786
27	42.864222	32.193560	27.270000	21.522899

MODEL E

JOINT GERERALIZED LEAST SQUARES

PREDICTION INTERVAL VALUES

LEAST SQUARES COVARIANCE MATRIX

0.11C79D 00	-0.39623D-C4	0.18428D-02	-0.24873D-01	-0.27123D 00
-0.39623D-C4	C.15334D-C7	-0.84344D-06	0.96256D-05	0.97001D-04
0.18428D-C2	-C.84344D-06	0.15326D-02	-0.15133D-01	0.281190-C2
-0.24873D-C1	0.96256D-C5	-0.15133D-01	0.17271D CC	-0.37743D-01
-0.27123D 00	C.97001D-04	0.28119D-02	-0.37743D-01	0.11792D 02
C.97001D-04	-0.37538D-07	-0.13039D-05	0.14881D-C4	-0.51080D-02
0.38512D-15	-0.1455CD-18	0.12907D-03	-0.14730D-02	0.42552D 01
-0.14794D 00	C.53053D-C4	-0.17310D-02	0.24036D-01	0.20186D 01
0.13400D-C3	-0.51855D-C7	0.21069D-05	-0.24045D-04	-0.18106D-02
-0.16806D-05	0.65C39D-C9	-0.11017D-06	0.12573D-05	-0.11404D-04

C.97001D-04	0.56295D-15	-0.14794D C0	0.13400D-03	-0.16806D-05
-0.37538D-07	-0.20522D-18	0.53053D-C4	-0.51855D-07	0.65C39D-09
-0.13039D-05	0.12907D-03	-0.17310D-C2	0.21069D-05	-0.11017D-06
0.14881D-04	-0.14730D-02	0.24036D-C1	-0.24045D-04	0.12573D-05
-0.51080D-02	0.42552D 01	0.20186D C1	-C.18106D-02	-C.114C4D-04
0.24721D-05	-0.26583D-02	-0.73074D-03	0.70197D-06	0.125C6D-07
-0.26583D-02	0.54287D 01	0.32746D-C1	-0.69297D-05	-0.43433D-04
-0.73074D-03	0.32746D-01	0.23744D 01	-0.212C2D-02	-0.86669D-04
0.70197D-06	-0.69297D-05	-0.21202D-C2	0.20499D-05	0.13304D-07
0.12506D-07	-0.43433D-04	-0.86669D-C4	0.133C4D-07	C.12376D-06

TABLE OF LS CORRELATED PREDICTION INTEPVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	42.996416	31.976923	27.270000	20.957429
2	38.402254	27.510643	28.390000	16.618931
3	38.815021	27.921190	28.400000	17.027360
4	53.056183	42.264582	51.610000	31.472982
5	48.544981	37.866001	40.040000	27.187021
6	48.133818	37.468809	39.150000	26.803801
7	47.942283	37.272837	41.020000	26.603392
8	48.133818	37.468809	38.200000	26.803801
9	48.645678	37.966762	34.570000	27.287847
10	48.133818	37.468809	35.910000	26.803801
11	48.133818	37.468809	50.980000	26.803801
12	48.133818	37.468809	37.340000	26.803801
13	48.908250	38.222020	37.320000	27.535791
14	48.921407	38.233249	33.690000	27.545092
15	24.942192	13.883649	11.320000	2.825107
16	23.050856	11.994706	11.510000	0.938556
17	23.050856	11.994706	12.080000	0.938556
18	23.050856	11.994706	12.490000	0.938556
19	22.225079	11.221966	8.700000	0.218853
20	20.082272	9.084766	12.930000	-1.912739
21	28.220243	17.501517	18.490000	6.782790
22	39.242649	28.342673	25.090000	17.442697
23	34.622194	23.856676	23.990000	13.091159
24	34.699258	23.941518	44.700000	13.183779
25	34.658616	23.902926	20.250000	13.147237
26	46.575587	35.767393	38.490000	24.959200
27	42.711111	31.986247	27.270000	21.261384

TABLE OF LS (INDEPEND) PREDICTION INTERVAL VALUES

	PI +	PREDICTED	ACTUAL	PI -
1	42.668549	31.974792	27.269989	21.281021
2	38.081787	27.508209	28.389999	16.934616
3	38.517990	27.918762	28.399994	17.319519
4	52.661728	42.262314	51.609985	31.862885
5	48.105057	37.863342	40.029993	27.621613
6	47.700073	37.466171	39.149994	27.232254
7	47.505875	37.270248	41.019989	27.034607
8	47.700073	37.466171	38.199997	27.232254
9	48.207306	37.964081	34.569992	27.720840
10	47.700073	37.466171	35.909988	27.232254
11	47.700073	37.466171	50.979996	27.232254
12	47.700073	37.466171	37.339996	27.232254
13	48.468903	38.219299	37.319992	27.969681
14	48.479965	38.230545	33.689987	27.981110
15	24.318436	13.882758	11.320000	3.447076
16	22.433441	11.993677	11.509999	1.553909
17	22.433441	11.993677	12.080000	1.553909
18	22.433441	11.993677	12.490000	1.553909
19	21.518036	11.221060	8.700000	0.924072
20	19.377182	9.083716	12.929999	-1.209761
21	27.563980	17.500229	18.489990	7.436467
22	38.695038	28.340729	25.089996	17.986404
23	34.076553	23.854492	23.989990	13.632421
24	34.152878	23.939346	44.699997	13.725811
25	34.112030	23.900742	20.250000	13.689447
26	46.046143	35.765503	38.489990	25.484848
27	42.138733	31.984024	27.269989	21.829300

APPENDIX D

COMPUTER PROGRAMS

A. JOINT GENERALIZED LEAST SQUARES PROGRAM

1. Program Description

The joint generalized least squares computer program is designed to read in a set of data and then carry out joint generalized least squares computations as specified on a control card, and selection cards. The program uses a number of IBM 360 Scientific Subroutine packages to do the calculations. It is programmed using FORTRAN IV language much like the Multiple Regression program in the scientific subroutine package (REGRE). The program conducts a number of standard multiple linear regressions (one for each subsystem CER). The residuals from the single regresions are saved and utilized to compute a matrix of correlations between selections. Both of these items are listed in the output. Next the program does the joint generalized least squares calculations; initially S, the matrix of mean squares and products of the least squares residuals, is computed, next b_j, the joint parameter estimate vector is computed along with $V(b_j)$, the covariance matrix of the joint estimates. Finally, a set of joint generalized least squares prediction interval values is calculated. The program was designed to be easy to use for analysis utilizing the joint generalized least squares method. Double precision was used to improve accuracy

by reducing roundoff errors. The program will handle up to thirty-six observations and nine subsystem relations. Each subsystem CER may have up to three explanatory variables. The program takes 280 K of core storage space to run with a maximum run time of about thirty seconds. It should be noted that larger dimensional problems may be possible but computer capacity rapidly becomes a constraint to be concerned with. The scientific subroutine ARRAY was used to allow variable dimensioning so that those subroutines that deal with matrices can operate on any size array, limited only by the maximum program dimensions.

2. JGLS Program Users Guide

The program listed herein is straightforward to use. The user must supply as input a data deck of independent and dependent variables. The data array is presently programed to store up to forty-one variables and up to thirty-six observations for each variable. The user should supply the appropriate read statements in the program to accommodate the data format he is using. After the data is read in any transformations desired may be made.

a. Control Card

One control card is required for each program and is read before the data deck. This card is prepared as follows:

COLUMNS	CONTENTS	SAMPLE
1-6	Problem Name	Escort
7-11	Number of Observations	36
12-13	Number of Variables	41
14-15	Number of Selection Cards	04

b. Selection Card

The selection card is used to specify the type of transformation desired, the dependent variable and a set of independent variables in a subsystem multiple linear regression analysis. One selection card is required for each subsystem in the model. The variables are designated as columns in the data array. The selection card is prepared as follows:

COLUMNS	CONTENTS	SAMPLE
1-2	Transformation Code 00 Linear 01 Log Linear	00
3-4	Dependent Variable Designated for the Regression	37
5-6	Number of Independent Variables Included in Regression	01
7-8	1st Independent Variable Included	21
9-10	2nd Independent Variable Included	
11-12	3rd Independent Variable Included	

The input format 36I2 is used for the selection card.

c. Deck Setup

The deck setup to run the program for a number of selections is as follows:

1. Main Program
2. Control Card
3. Data deck (Read statements must be supplied in the main program.
4. Selection Cards (One selection card for each subsystem multiple linear regression).

The joint generalized least squares program is listed in the following section of this Appendix.

```
C  PROGRAM TO COMPUTE LEAST SQUARES COST ESTIMATING RELATIONSHIPS FOR A
C  NUMBER OF DESIGNATED SELECTIONS. THE RESIDUAL VALUES FROM THESE
C  SELECTIONS ARE THEN USED IN THE COMPUTATION OF JOINT GENERALIZED LEAST
C  SQUARES ESTIMATES AND A COVARIANCE MATRIX OF THE JOINT ESTIMATES.
C  FINALLY, PREDICTION INTERVAL VALUES ARE COMPUTED FOR THE GIVEN OBSER-
C  VATIONS UTILIZING THE JOINT GENERALIZED LEAST SQUARES ESTIMATES.
C
C  OUTPUT FOR THE PROGRAM CONSISTS OF:
C     1. INDIVIDUAL REGRESSION OUTPUT
C     2. LISTING OF RESIDUALS FOR THE SELECTIONS
C     3. MATRIX OF CORRELATIONS BETWEEN SELECTIONS
C     4. COVARIANCE MATRIX OF THE JOINT ESTIMATES
C     5. JOINT GENERALIZED LEAST SQUARES ESTIMATES
C     6. TABLE OF JGLS PREDICTION INTERVAL VALUES
C
C  USER MUST SUPPLY DATA INPUT AND CORRESPONDING READ STATEMENTS IN THE
C  MAIN PROGRAM. ALSO A CONTROL CARD AND SELECTION CARDS MUST BE PROVIDED
C  FOR THE REGRESSIONS DESIRED AS DESCRIBED IN THE USER'S GUIDE.
C
C
// EXEC FORTCLG,REGION.GO=350K
//FORT.SYSIN DD *
       IMPLICIT REAL*8(A-H,O-V,X-Z), INTEGER*4(W)
       DIMENSION DATA(36,41)
       DIMENSION Y(36), X(36,4), XT(4,36), XTX(4,4), W1(4), W2(4),
      1XTXXT(4,36), B(4), ISAVE(10), XB(36), RESID(36), SS(1), YY(1),
      2RESIDM(36,9)
       DIMENSION YGLS(324), XGLS(324,36), BBB(36)
       DIMENSION COMP1(36,324), COMP2(36,36), BB(36), E(9,9), W3(9),
      1W4(9), W5(36), W6(36)
       DIMENSION SSAVE(9), ROE(9,9), NSAVE(9)
       DIMENSION XOBS(36), CHAT(1), XSAVE(36), VSAVE(1), PIP(36), PIM(36)
      1,PRED(36)
       DATA NNI/0/, NNIG/0/, NN/0/
       DATA XGLS/11664*0.0/, COMP1/11664*0.0/
       DATA XOBS/36*0.0/
C   READ PROBLEM PARAMETER CARD
       READ(5,1)PR, PR1, N, M, NS
     1 FORMAT(A4,A2,I5,2I2)
C
C  PR,PR1....PROBLEM NUMBER
C   N.........NUMBER OF OBSERVATIONS
C   M.........NUMBER OF VARIABLES
C   NS.......NUMBER OF SELECTIONS
C
C   OBSERVATION MATRIX READ IN
       DO 6000 I=1,36
       READ(5,6010)(DATA(I,J),J=1,36)
```

```
 6000 CONTINUE
 6010 FORMAT(/12F6.0/12F6.0/5F6.0/7F6.0)
      READ(5,6020)(DATA(I,37),I=1,36)
      READ(5,6020)(DATA(I,38),I=1,36)
      READ(5,6020)(DATA(I,39),I=1,36)
      READ(5,6020)(DATA(I,40),I=1,36)
      READ(5,6020)(DATA(I,41),I=1,36)
 6020 FORMAT(9F8.0)
C
C   TEST NUMBER OF SELECTIONS
      IF(NS) 108,108,109
  108 WRITE(6,4)
    4 FORMAT(53H1NUMBER OF SELECTIONS NOT SPECIFIED.  JOB TERMINATED.)
      GO TO 300
C
C   COMMENCE MULTIPLE REGRESSION PROGRAM
  109 DO 300 I=1,NS
      WRITE(6,5)PR,PR1,I
    5 FORMAT(25H1MULTIPLE REGRESSION.....A4,A2//6X,14HSELECTION.....I2//
     1)
C   READ IN SUBSET SELECTION CARD
C
      READ(5,3)NT,NDEP,NI,(ISAVE(J),J=1,NI)
    3 FORMAT(36I2)
      WRITE(6,6001)NT,NDEP,NI,(ISAVE(J),J=1,NI)
 6001 FORMAT('0','SELECTION CARD',15I2)
C
C   NT.......MODEL TRANSFORMATION CODE DESIRED
C      0.....LINEAR
C      1.....LOG LINEAR
C   NI.......NUMBER OF INDEPENDENT VARIABLES INCLUDED
C   NDEP.....DEPENDENT VARIABLE
C   ISAVE....AVECTOR CONTAINING THE INDEPENDENT VARIABLES INCLUDED
C
C   TRANSFORM DATA AND LOAD THE X AND Y ARRAYS
      DO 110 J=1,N
      Y(J)=DATA(J,NDEP)
      IF(NT) 113, 112, 113
  112 GO TO 114
  113 Y(J)=DLOG(Y(J))
      GO TO 114
  114 YGLS(NN+J)=Y(J)
      X(J,1)=1.0
      XGLS(NN+J,NNIG+1)=1.0
  110 CONTINUE
      DO 120 J=1,NI
      MM=ISAVE(J)
      DO 130 JJ=1,N
```

```
      X(JJ,J+1)=DATA(JJ,MM)
  118 XGLS(NN+JJ,NNIG+J+1)=X(JJ,J+1)
  130 CONTINUE
  120 CONTINUE
      NNI=NI+1
      NSAVE(I)=NNI
      NNIG=NNIG+NNI
      NN=NN+N
C
C  CALCULATE VECTOR OF ESTIMATES
      CALL ARRAY(2,N,NNI,36,4,X,X)
      CALL ARRAY(2,NNI,NNI,4,4,XTX,XTX)
      CALL ARRAY(2,NNI,N,4,36,XT,XT)
      CALL ARRAY(2,NNI,N,4,36,XTXXT,XTXXT)
      CALL GTPRD(X,X,XTX,N,NNI,NNI)
      CALL DMINV(XTX,NNI,DET,W1,W2)
      CALL GMTRA(X,XT,N,NNI)
      CALL GMPRD(XTX,XT,XTXXT,NNI,NNI,N)
      CALL GMPRD(XTXXT,Y,B,NNI,N,1)
      WRITE(6,5040)
 5040 FORMAT('0',T10,'LEAST SQUARES REGRESSION COEFICIENTS'//)
      WRITE(6,6)(B(J),J=1,NNI)
    6 FORMAT(' ',10F12.6)
C
C  CALCULATE RESIDUAL VECTOR
      CALL GMPRD(X,B,XB,N,NNI,1)
      CALL GMSUB(Y,XB,RESID,N,1)
C
C  ENTER RESIDUALS INTO RESIDUAL MATRIX
      DO 290 J=1,N
      RESIDM(J,I)=RESID(J)
  290 CONTINUE
C
C  CALCULATE THE STANDARD ERROR OF THE ESTIMATE
  397 CALL GTPRD(RESID,RESID,SS,N,1,1)
      BN=DFLOAT(N-NNI)
      S1=SS(1)/BN
      S=DSQRT(S1)
      SSAVE(I)=S
      WRITE(6,8)S
    8 FORMAT('0','STD. ERROR OF ESTIMATE=  ',F13.6//)
C
C  CALCULATE THE COVARIANCE MATRIX OF THE ESTIMATES
      CALL ARRAY(1,NNI,NNI,4,4,XTX,XTX)
      DO 141 J=1,NNI
      DO 142 K=1,NNI
      XTX(J,K)=S1*XTX(J,K)
  142 CONTINUE
```

```
  141 CONTINUE
      WRITE(6,5050)
 5050 FORMAT('0',T10,'COVARIANCE MATRIX OF THE ESTIMATES'/)
      DO 143 J=1,NNI
      WRITE(6,144)(XTX(J,K),K=1,NNI)
  143 CONTINUE
  144 FORMAT(' ',2X,8D13.5)
C
C  CALCULATE ADJUSTED MULTIPLE CORRELATION COEFICIENT
  990 SUM=0.0
      DO 150 J=1,N
      SUM=SUM+Y(J)
  150 CONTINUE
  992 AN=DFLOAT(N)
      YBAR=SUM/AN
      DO 160 J=1,N
      Y(J)=Y(J)-YBAR
  160 CONTINUE
      CALL GTPRD(Y,Y,YY,N,1,1)
      CN=DFLOAT(N-1)
      R=1.0-(S1/(YY(1)/CN))
      WRITE(6,9)R
    9 FORMAT('0','ADJUSTED MULTIPLE CORRELATION COEFICIENT',F9.6)
  300 CONTINUE
      WRITE(6,5000)
 5000 FORMAT('1','MATRIX OF RESIDUALS FOR SELECTIONS'//2X,'CASE NO.',
     1T31,'SELECTION NUMBER'/18X,'1',T30,'2',T42,'3',T54,'4',T66,'5',
     2T78,'6',T90,'7',T102,'8',T114,'9'/)
      DO 291 J=1,N
      WRITE(6,292)J,(RESIDM(J,K),K=1,NS)
  291 CONTINUE
  292 FORMAT(' ',5X,I3,9F12.5)
C
CC
C  THE ESTIMATE FOR SIGMA IS COMPUTED
      CALL ARRAY(2,N,NS,36,9,RESIDM,RESIDM)
      CALL ARRAY(2,NS,NS,9,9,E,E)
      CALL GTPRD(RESIDM,RESIDM,E,N,NS,NS)
      CALL ARRAY(1,NS,NS,9,9,E,E)
      DO 1000 I=1,NS
      DO 1010 J=1,NS
      E(I,J)=E(I,J)/AN
 1010 CONTINUE
 1000 CONTINUE
      SSUM=0.0
      DO 1001 I=1,NS
      DO 1011 J=1,NS
      ROE(I,J)=E(I,J)/(DSQRT(E(I,I)*E(J,J)))
```

```
      SSUM=SSUM+E(I,J)
 1011 CONTINUE
 1001 CONTINUE
      WRITE(6,6250)
 6250 FORMAT('1','MATRIX OF CORRELATIONS BETWEEN SELECTIONS'//1X,
     1'SELECTION NO.',5X,'1',T30,'2',T42,'3',T54,'4',T66,'5',T78,'6',
     2T90,'7',T102,'8',T114,'9'//)
      DO 6300 I=1,NS
      WRITE(6,6400)I,(ROE(I,J),J=1,NS)
 6300 CONTINUE
 6400 FORMAT(' ',5X,I3,9F12.6)
      WRITE(6,5010)
 5010 FORMAT('1','JOINT GENERALIZED LEAST SQUARES REGRESSION OUTPUT'/)
C
C  THE ESTIMATE FOR SIGMA IS INVERTED
      CALL ARRAY(2,NS,NS,9,9,E,E)
      CALL DMINV(E,NS,DET,W3,W4)
      CALL ARRAY(1,NS,NS,9,9,E,E)
C
C
C  COMPUTE X TRANSPOSE TIMES THE KRONECKER PRODUCT OF SIGMA INVERSE AND
C  THE IDENTITY MATRIX
      KKK=((N*(NS-1))+1)
      NEC=1
      NC=N+1
      JJ=0
      DO 900 J=1,NN
      JJ=JJ+1
      IF(J.NE.NC) GO TO 910
      JJ=1
      NEC=NEC+1
      NC=((NEC*N)+1)
  910 DO 920 I=1,NNIG
      NER=0
      CSAVE=0.0
      DO 930 M=1,KKK,N
      NER=NER+1
      CSAVE=CSAVE+XGLS(JJ+M-1,I)*E(NER,NEC)
  930 CONTINUE
      COMP1(I,J)=CSAVE
  920 CONTINUE
  900 CONTINUE
C
C  MATRIX MANIPULATIONS USED TO COMPUTE JOINT VECTOR
C  OF ESTIMATES
      CALL ARRAY(2,NN,NNIG,324,36,XGLS,XGLS)
      CALL ARRAY(2,NNIG,NN,36,324,COMP1,COMP1)
      CALL ARRAY(2,NNIG,NNIG,36,36,COMP2,COMP2)
```

```
      CALL GMPRD(COMP1,YGLS,BB,NNIG,NN,1)
      CALL GMPRD(COMP1,XGLS,CCMP2,NNIG,NN,NNIG)
      CALL DMINV(COMP2,NNIG,DET,W5,W6)
      CALL GMPRD(COMP2,BB,BBB,NNIG,NNIG,1)
      CALL ARRAY(1,NNIG,NNIG,36,36,COMP2,COMP2)
      WRITE(6,1091)
 1091 FORMAT(' ',5X,'COVARIANCE MATRIX OF THE JCINT ESTIMATES'/)
      IF(10-NNIG) 6500,6450,6450
 6450 DC 2000 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=1,NNIG)
 2000 CCNTINUE
 1071 FCRMAT(' ',2X,10D13.5)
      GC TO 6600
 6500 Z=1.0
      IF(20-NNIG) 6520,6510,6510
 6510 Z=1.0
      DC 6530 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=1,10)
 6530 CONTINUE
      WRITE(6,6561)
 6561 FCRMAT(' ')
      DC 6540 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=11,NNIG)
 6540 CCNTINUE
      GO TO 6600
 6520 Z=1.0
      DC 6550 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=1,10)
 6550 CCNTINUE
      WRITE(6,6561)
      DC 6560 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=11,20)
 6560 CCNTINUE
      WRITE(6,6561)
      DC 6570 I=1,NNIG
      WRITE(6,1071)(COMP2(I,J),J=21,NNIG)
 6570 CONTINUE
 6600 WRITE(6,5030)DET
 5030 FORMAT(' ','COMPUTATICNAL DETERMINANT CHECK=  ',E13.7//)
C
C  THE ESTIMATES ARE READ OUT
      WRITE(6,2011)
 2011 FCRMAT('1',T10,'JCINT LEAST SQUARES ESTIMATES'/' ',2X,'SELECTICN
     1NO.',20X,'COEFICIENTS'/' ',T20,'CONSTANT',T35,'B1',T48,'B2',T61,
     2'B3'/)
      LL=1
      DC 2013 I=1,NS
      LM=LL+NSAVE(I)-1
```

```
      WRITE(6,2012)I,(BBB(J),J=LL,LM)
      LL=LL+NSAVE(I)
 2013 CONTINUE
 2012 FORMAT('0',6X,I3,3X,5F13.6/)
C
C  CALCULATE PREDICTION INTERVAL
C
      CALL ARRAY(2,NNIG,NNIG,36,36,COMP2,COMP2)
      CALL ARRAY(1,NN,NNIG,324,36,XGLS,XGLS)
      DO 9100 I=1,N
      NSS=1
      NSSS=0
      L=0
      DO 9200 J=1,KKK,N
      L=L+1
      NSSS=NSSS+NSAVE(L)
      DO 9300 K=NSS,NSSS
      XOBS(K)=XGLS(I+J-1,K)
 9300 CONTINUE
      NSS=NSS+NSAVE(L)
 9200 CONTINUE
      CALL GTPRD(XOBS,BBB,CHAT,NNIG,1,1)
      PRED(I)=CHAT(1)
      CALL GMPRD(COMP2,XOBS,XSAVE,NNIG,NNIG,1)
      CALL GTPRD(XOBS,XSAVE,VSAVE,NNIG,1,1)
      SSSUM=1.96*DSQRT( SSUM+VSAVE(1))
      PIP(I)=CHAT(1)+SSSUM
      PIM(I)=CHAT(1)-SSSUM
 9100 CONTINUE
      WRITE(6,9005)
 9005 FORMAT('1',//////////19X,'TABLE OF PREDICTION INTERVAL VALUES'/'0'
     1,17X,'PI +',9X,'PREDICTED',4X,'ACTUAL',7X,'PI -'/'0')
      DO 9000 I=1,N
      XSAVE(I)=DATA(I,36)
      WRITE(6,9010)I,PIP(I),PRED(I),XSAVE(I),PIM(I)
 9000 CONTINUE
 9010 FORMAT(' ',11X,I2,4F13.6)
      STOP
      END


      SUBROUTINE ARRAY (MODE,I,J,N,M,S,D)
      DOUBLE PRECISION S,D
      DIMENSION S(1),D(1)
      NI=N-I
      IF(MODE-1) 100, 100, 120
  100 IJ=I*J+1
      NM=N*J+1
```

```
      DO 110 K=1,J
      NM=NM-NI
      DO 110 L=1,I
      IJ=IJ-1
      NM=NM-1
  110 D(NM)=S(IJ)
      GO TO 140
  120 IJ=0
      NM=0
      DO 130 K=1,J
      DO 125 L=1,I
      IJ=IJ+1
      NM=NM+1
  125 S(IJ)=D(NM)
  130 NM=NM+NI
  140 RETURN
      END

      SUBROUTINE GTPRD(A,B,R,N,M,L)                                    GTPR 360
      DOUBLE PRECISION A,B,R
      DIMENSION A(1),B(1),R(1)                                         GTPR 370
C                                                                      GTPR 380
      IR=0                                                             GTPR 390
      IK=-N                                                            GTPR 400
      DO 10 K=1,L                                                      GTPR 410
      IJ=0                                                             GTPR 420
      IK=IK+N                                                          GTPR 430
      DO 10 J=1,M                                                      GTPR 440
      IB=IK                                                            GTPR 450
      IR=IR+1                                                          GTPR 460
      R(IR)=0                                                          GTPR 470
      DO 10 I=1,N                                                      GTPR 480
      IJ=IJ+1                                                          GTPR 490
      IB=IB+1                                                          GTPR 500
   10 R(IR)=R(IR)+A(IJ)*B(IB)                                          GTPR 510
      RETURN                                                           GTPR 520
      END                                                              GTPR 530

      SUBROUTINE GMSUB(A,B,R,N,M)                                      GMSU 320
      DOUBLE PRECISION A,B,R
      DIMENSION A(1),B(1),R(1)                                         GMSU 330
C                                                                      GMSU 340
C        CALCULATE NUMBER OF ELEMENTS                                  GMSU 350
C                                                                      GMSU 360
      NM=N*M                                                           GMSU 370
C                                                                      GMSU 380
```

```
C          SUBTRACT MATRICES                                          GMSU 390
C                                                                     GMSU 400
      DO 10 I=1,NM                                                    GMSU 410
   10 R(I)=A(I)-B(I)                                                  GMSU 420
      RETURN                                                          GMSU 430
      END                                                             GMSU 440


      SUBROUTINE GMPRD(A,B,R,N,M,L)                                   GMPR 370
      DOUBLE PRECISION A,B,R
      DIMENSION A(1),B(1),R(1)                                        GMPR 380
C                                                                     GMPR 390
      IR=0                                                            GMPR 400
      IK=-M                                                           GMPR 410
      DO 10 K=1,L                                                     GMPR 420
      IK=IK+M                                                         GMPR 430
      DO 10 J=1,N                                                     GMPR 440
      IR=IR+1                                                         GMPR 450
      JI=J-N                                                          GMPR 460
      IB=IK                                                           GMPR 470
      R(IR)=0                                                         GMPR 480
      DO 10 I=1,M                                                     GMPR 490
      JI=JI+N                                                         GMPR 500
      IB=IB+1                                                         GMPR 510
   10 R(IR)=R(IR)+A(JI)*B(IB)                                         GMPR 520
      RETURN                                                          GMPR 530
      END                                                             GMPR 540


      SUBROUTINE GMTRA(A,R,N,M)                                       GMTR 300
      DOUBLE PRECISION A,R
      DIMENSION A(1),R(1)                                             GMTR 310
C                                                                     GMTR 320
      IR=0                                                            GMTR 330
      DO 10 I=1,N                                                     GMTR 340
      IJ=I-N                                                          GMTR 350
      DO 10 J=1,M                                                     GMTR 360
      IJ=IJ+N                                                         GMTR 370
      IR=IR+1                                                         GMTR 380
   10 R(IR)=A(IJ)                                                     GMTR 390
      RETURN                                                          GMTR 400
      END                                                             GMTR 410
```

```
C PROGRAM TO COMPUTE A COVARIANCE MATRIX OF THE LEAST SQUARES REGRESSION
C COEFICIENT ESTIMATES UNDER THE ASSUMPTION OF CORRELATIONS BETWEEN IND-
C IVIDUAL SELECTIONS. NEXT THE PROGRAM UTILIZES THIS COVARIANCE MATRIX
C TO COMPUTE LEAST SQUARES CORRELATED PREDICTION INTERVALS FOR THE GIVEN
C CBSERVATIONS.
C
C
// EXEC FORTCLG,REGION.GO=100K
//FORT.SYSIN DD *
      IMPLICIT REAL*8(A-H,O-V,X-Z), INTEGER*4(W)
      DIMENSION DATA(36,41)
      DIMENSION X1(36,4),X2(36,4),ISAVE(10),X1TX1(4,4),W1(4),W2(4),
     1COMP1(4,4),X2TX2(4,4),COMP2(4,4),COMP3(4,4)
      DIMENSION XBIG(36,30)
      DIMENSION CVARB(30,30)
      DIMENSION XOBS(36),CHAT(1),XSAVE(36),VSAVE(1),
     1PIP(36),PIM(36),PRED(36),B(36)
      DATA MMI/0/, MSI/0/
      DATA XBIG/1080*0.0/
      DATA NNI/0/, NSI/0/
C READ PROBLEM PARAMETER CARD
      READ(5,1)PR,PR1,N,M,NS
    1 FORMAT(A4,A2,I5,2I2)
C READ IN OBSERVATION MATRIX
      DO 6000 I=1,36
      READ(5,6010)(DATA(I,J),J=1,36)
 6000 CONTINUE
 6010 FORMAT(/12F6.0/12F6.0/5F6.0/7F6.0)
      READ(5,6020)(DATA(I,37),I=1,36)
      READ(5,6020)(DATA(I,38),I=1,36)
      READ(5,6020)(DATA(I,39),I=1,36)
      READ(5,6020)(DATA(I,40),I=1,36)
      READ(5,6020)(DATA(I,41),I=1,36)
 6020 FORMAT(9F8.0)
C READ IN FIRST SUBSET SELECTION CARD
      DO 305 JK=1,NS
      READ(5,3)NT,NDEP,NI,(ISAVE(J),J=1,NI)
    3 FORMAT(36I2)
      DO 110 J=1,N
      X1(J,1)=1.0D0
  110 CONTINUE
      DO 120 J=1,NI
      MM=ISAVE(J)
      DO 130 JJ=1,N
      X1(JJ,J+1)=DATA(JJ,MM)
  130 CONTINUE
  120 CONTINUE
      NNI=NI+1
```

```
      CALL ARRAY(2,N,NNI,36,4,X1,X1)
      CALL GTPRD(X1,X1,X1TX1,N,NNI,NNI)
      CALL DMINV(X1TX1,NNI,DET,W1,W2)
C READ IN SECOND SUBSET SELECTION CARD
      NNIG=0
      DO 500 JJJ=1,NS
      READ(5,3)NT,NDEP,NI,(ISAVE(J),J=1,NI)
      DO 200 J=1,N
      X2(J,1)=1.0D0
      IF(JK.NE.1) GO TO 200
      XBIG(J,NNIG+1)=1.0D0
  200 CONTINUE
      DO 210 J=1,NI
      MM=ISAVE(J)
      DO 230 JJ=1,N
      X2(JJ,J+1)=DATA(JJ,MM)
      IF(JK.NE.1) GO TO 230
      XBIG(JJ,NNIG+J+1)=X2(JJ,J+1)
  230 CONTINUE
  210 CONTINUE
      NSI=NI+1
      NNIG=NNIG+NSI
C READ IN COVARIANCE ESTINATE
      READ(5,25)S
   25 FORMAT(F10.0)
      IF(JK.NE.JJJ) GO TO 600
      CALL ARRAY(2,N,NSI,36,4,X2,X2)
      CALL GTPRD(X2,X2,COMP3,N,NSI,NSI)
      CALL DMINV(COMP3,NSI,DET,W1,W2)
      GO TO 610
  600 CALL ARRAY(2,N,NSI,36,4,X2,X2)
      CALL GTPRD(X1,X2,COMP1,N,NNI,NSI)
      CALL GTPRD(X2,X2,X2TX2,N,NSI,NSI)
      CALL DMINV(X2TX2,NSI,DET,W1,W2)
      CALL GMPRD(X1TX1,COMP1,COMP2,NNI,NNI,NSI)
      CALL GMPRD(COMP2,X2TX2,COMP3,NNI,NSI,NSI)
  610 CALL ARRAY(1,NNI,NSI,4,4,COMP3,COMP3)
      DO 300 K=1,NNI
      DO 310 L=1,NSI
      COMP3(K,L)=S*COMP3(K,L)
  310 CONTINUE
  300 CONTINUE
      WRITE(6,10)
   10 FORMAT('0',3X,'OUTPUT')
      DO 320 K=1,NNI
      WRITE(6,11)(COMP3(K,J),J=1,NSI)
  320 CONTINUE
   11 FORMAT(' ',3X,4F13.6)
```

```
      DO 620 K=1,NNI
      DO 630 L=1,NSI
      CVARB(MMI+K,MSI+L)=COMP3(K,L)
  630 CONTINUE
  620 CONTINUE
      MSI=MSI+NSI
  500 CONTINUE
      MMI=MMI+NNI
      MSI=0
  305 CONTINUE
      DO 304 I=1,N
      WRITE(6,303)(XBIG(I,J),J=1,12)
  304 CONTINUE
  303 FORMAT(' ',2X,12F10.2)
      WRITE(6,1070)
 1070 FORMAT('1',5X,'LEAST SQUARES COVARIANCE MATRIX'/)
      IF(10-NNIG) 6500,6450,6450
 6450 DO 2000 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=1,NNIG)
 2000 CONTINUE
 1071 FORMAT(' ',2X,10D13.5)
      GO TO 6600
 6500 Z=1.0
      IF(20-NNIG) 6520,6510,6510
 6510 Z=1.0
      DO 6530 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=1,10)
 6530 CONTINUE
      WRITE(6,6561)
 6561 FORMAT(' ')
      DO 6540 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=11,NNIG)
 6540 CONTINUE
      GO TO 6600
 6520 Z=1.0
      DO 6550 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=1,10)
 6550 CONTINUE
      WRITE(6,6561)
      DO 6560 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=11,20)
 6560 CONTINUE
      WRITE(6,6561)
      DO 6570 I=1,NNIG
      WRITE(6,1071)(CVARB(I,J),J=21,NNIG)
 6570 CONTINUE
 6600 READ(5,632)(B(J),J=1,NNIG)
  632 FORMAT(8F10.0)
```

```
      READ(5,631)BSSUM
  631 FORMAT(F10.0)
      CALL ARRAY(2,NNIG,NNIG,30,30,CVARB,CVARB)
      DO 9100 I=1,N
      DO 9300 K=1,NNIG
      XOBS(K)=XBIG(I,K)
 9300 CONTINUE
      CALL GTPRD(XOBS,B,CHAT,NNIG,1,1)
      PRED(I)=CHAT(1)
      CALL GMPRD(CVARB,XOBS,XSAVE,NNIG,NNIG,1)
      CALL GTPRD(XOBS,XSAVE,VSAVE,NNIG,1,1)
      SSSUM=1.96*DSQRT(BSSUM+VSAVE(1))
      PIP(I)=CHAT(1)+SSSUM
      PIM(I)=CHAT(1)-SSSUM
 9100 CONTINUE
      WRITE(6,9005)
 9005 FORMAT('1'/////////16X,'TABLE OF 95 CORRELATED PREDICTION INTERVA
     1L VALUES'/'0',17X,'PI +',9X,'PREDICTED',4X,'ACTUAL',7X,'PI -'/'0')
      DO 9000 I=1,N
      XSAVE(I)=DATA(I,36)
      WRITE(6,9010)I,PIP(I),PRED(I),XSAVE(I),PIM(I)
 9000 CONTINUE
 9010 FORMAT(' ',11X,I2,4F13.6)
      STOP
      END


      SUBROUTINE ARRAY (MODE,I,J,N,M,S,D)
      DOUBLE PRECISION S,D
      DIMENSION S(1),D(1)
      NI=N-I
      IF(MODE-1) 100, 100, 120
  100 IJ=I*J+1
      NM=N*J+1
      DO 110 K=1,J
      NM=NM-NI
      DO 110 L=1,I
      IJ=IJ-1
      NM=NM-1
  110 D(NM)=S(IJ)
      GO TO 140
  120 IJ=0
      NM=0
      DO 130 K=1,J
      DO 125 L=1,I
      IJ=IJ+1
      NM=NM+1
  125 S(IJ)=D(NM)
```

```
  130 NM=NM+NI
  140 RETURN
      END


      SUBROUTINE GTPRD(A,B,R,N,M,L)                                    GTPR 360
      DOUBLE PRECISION A,B,R
      DIMENSION A(1),B(1),R(1)                                         GTPR 370
C                                                                      GTPR 380
      IR=0                                                             GTPR 390
      IK=-N                                                            GTPR 400
      DO 10 K=1,L                                                      GTPR 410
      IJ=0                                                             GTPR 420
      IK=IK+N                                                          GTPR 430
      DO 10 J=1,M                                                      GTPR 440
      IB=IK                                                            GTPR 450
      IR=IR+1                                                          GTPR 460
      R(IR)=0                                                          GTPR 470
      DO 10 I=1,N                                                      GTPR 480
      IJ=IJ+1                                                          GTPR 490
      IB=IB+1                                                          GTPR 500
   10 R(IR)=R(IR)+A(IJ)*B(IB)                                          GTPR 510
      RETURN                                                           GTPR 520
      END                                                              GTPR 530


      SUBROUTINE GMSUB(A,B,R,N,M)                                      GMSU 320
      DOUBLE PRECISION A,B,R
      DIMENSION A(1),B(1),R(1)                                         GMSU 330
C                                                                      GMSU 340
CCC      CALCULATE NUMBER OF ELEMENTS                                  GMSU 350
CCC                                                                    GMSU 360
      NM=N*M                                                           GMSU 370
C                                                                      GMSU 380
CCC      SUBTRACT MATRICES                                             GMSU 390
CCC                                                                    GMSU 400
      DO 10 I=1,NM                                                     GMSU 410
   10 R(I)=A(I)-B(I)                                                   GMSU 420
      RETURN                                                           GMSU 430
      END                                                              GMSU 440


      SUBROUTINE GMPRD(A,B,R,N,M,L)                                    GMPR 370
      DOUBLE PRECISION A,B,R
      DIMENSION A(1),B(1),R(1)                                         GMPR 380
C                                                                      GMPR 390
      IR=0                                                             GMPR 400
      IK=-M                                                            GMPR 410
```

```
      DO 10 K=1,L                                                       GMPR 420
      IK=IK+M                                                           GMPR 430
      DO 10 J=1,N                                                       GMPR 440
      IR=IR+1                                                           GMPR 450
      JI=J-N                                                            GMPR 460
      IB=IK                                                             GMPR 470
      R(IR)=0                                                           GMPR 480
      DO 10 I=1,M                                                       GMPR 490
      JI=JI+N                                                           GMPR 500
      IB=IB+1                                                           GMPR 510
   10 R(IR)=R(IR)+A(JI)*B(IB)                                           GMPR 520
      RETURN                                                            GMPR 530
      END                                                               GMPR 540

      SUBROUTINE GMTRA(A,R,N,M)                                         GMTR 300
      DOUBLE PRECISION A,R
      DIMENSION A(1),R(1)                                               GMTR 310
C                                                                       GMTR 320
      IR=0                                                              GMTR 330
      DO 10 I=1,N                                                       GMTR 340
      IJ=I-N                                                            GMTR 350
      DO 10 J=1,M                                                       GMTR 360
      IJ=IJ+N                                                           GMTR 370
      IR=IR+1                                                           GMTR 380
   10 R(IR)=A(IJ)                                                       GMTR 390
      RETURN                                                            GMTR 400
      END                                                               GMTR 410
```

```
C PROGRAM TO COMPUTE A NUMBER OF MULTIPLE LINEAR REGRESSIONS AND THEN TO
C COMPUTE A COMBINED LEAST SQUARES PREDICTION INTERVAL UNDER THE ASSUMP-
C TION OF INDEPENDENCE BETWEEN ALL SELECTIONS.
C
C
      DIMENSION DATA(36,41)
      DIMENSION Y(36), X(36,4), XT(4,36), XTX(4,4), W1(4), W2(4),
     1XTXXT(4,36), B(4), ISAVE(10), XB(36), RESID(36), SS(1), YY(1)
      DIMENSION SSAVE(9), ROE(9,9), NSAVE(9)
      DIMENSION YHAT(1),PSAVE(36),A(36),TCSAVE(36),PIP(36),PIM(36),
     1XOBS(4),PHAT(1)
      DATA NNI/0/, NNIG/0/, NN/0/
      DATA A/36*0.0/, TCSAVE/36*0.0/
C  READ PROBLEM PARAMETER CARD
      READ(5,1)PR, PR1, N, M, NS
    1 FORMAT(A4,A2,I5,2I2)
C
C PR,PR1....PROBLEM NUMBER
C  N.........NUMBER OF OBSERVATIONS
C  M.........NUMBER OF VARIABLES
C  NS........NUMBER OF SELECTIONS
C
C  OBSERVATION MATRIX READ IN
      DO 6000 I=1,36
      READ(5,6010)(DATA(I,J),J=1,36)
 6000 CONTINUE
 6010 FORMAT(/12F6.0/12F6.0/5F6.0/7F6.0)
      READ(5,6020)(DATA(I,37),I=1,36)
      READ(5,6020)(DATA(I,38),I=1,36)
      READ(5,6020)(DATA(I,39),I=1,36)
      READ(5,6020)(DATA(I,40),I=1,36)
      READ(5,6020)(DATA(I,41),I=1,36)
 6020 FORMAT(9F8.0)
C
C  NOTE  COL37 BASE CST, COL38 ENG CST, COL 39 PAYLOAD CST, COL 40 CONST CST
C  COL 41 ENG WGT
C
C  TEST NUMBER OF SELECTIONS
      IF(NS) 108,108,109
  108 WRITE(6,4)
    4 FORMAT(53H1NUMBER OF SELECTIONS NOT SPECIFIED.  JOB TERMINATED.)
      GO TO 300
C
C  COMMENCE MULTIPLE REGRESSION PROGRAM
  109 DO 300 I=1,NS
      WRITE(6,5)PR,PR1,I
    5 FORMAT(25H1MULTIPLE REGRESSION.....A4,A2//6X,14HSELECTION.....I2//
     1)
```

```
C
C  READ IN SUBSET SELECTION CARD
      READ(5,3)NT,NDEP,NI,(ISAVE(J),J=1,NI)
    3 FORMAT(36I2)
      WRITE(6,6001)NT,NDEP,NI,(ISAVE(J),J=1,NI)
 6001 FORMAT('0','SELECTION CARD',15I2)
C
C  NT.......MODEL TRANSFORMATION CODE DESIRED
C     0.....LINEAR
C     1.....LOG LINEAR
C  NDEP.....DEPENDENT VARIABLE
C  NI.......NUMBER OF INDEPENDENT VARIABLES INCLUDED
C  ISAVE....AVECTOR CONTAINING THE INDEPENDENT VARIABLES INCLUDED
C
C  TRANSFORM DATA AND LOAD THE X AND Y ARRAYS
      DO 110 J=1,N
      Y(J)=DATA(J,NDEP)
      IF(NT) 113, 112, 113
  112 GO TO 114
  113 Y(J)=ALOG(Y(J))
      GO TO 114
  114 X(J,1)=1.0
  110 CONTINUE
      DO 120 J=1,NI
      MM=ISAVE(J)
      DO 130 JJ=1,N
      X(JJ,J+1)=DATA(JJ,MM)
  130 CONTINUE
  120 CONTINUE
      NNI=NI+1
      NSAVE(I)=NNI
      NNIG=NNIG+NNI
      NN=NN+N
C
C  CALCULATE VECTOR OF ESTIMATES
      CALL ARRAY(2,N,NNI,36,4,X,X)
      CALL ARRAY(2,NNI,NNI,4,4,XTX,XTX)
      CALL ARRAY(2,NNI,N,4,36,XT,XT)
      CALL ARRAY(2,NNI,N,4,36,XTXXT,XTXXT)
      CALL GTPRD(X,X,XTX,N,NNI,NNI)
      CALL MINV(XTX,NNI,DET,W1,W2)
      CALL GMTRA(X,XT,N,NNI)
      CALL GMPRD(XTX,XT,XTXXT,NNI,NNI,N)
      CALL GMPRD(XTXXT,Y,B,NNI,N,1)
      WRITE(6,5040)
 5040 FORMAT('0',T10,'LEAST SQUARES REGRESSION COEFICIENTS'//)
      WRITE(6,6)(B(J),J=1,NNI)
    6 FORMAT(' ',10F12.6)
```

```
C
C  CALCULATE RESIDUAL VECTOR
      CALL GMPRD(X,B,XB,N,NNI,1)
      CALL GMSUB(Y,XB,RESID,N,1)
C
CC
C  CALCULATE THE STANDARD ERROR OF THE ESTIMATE
      CALL GTPRD(RESID,RESID,SS,N,1,1)
      BN=N-NNI
      S1=SS(1)/BN
      S=SQRT(S1)
      SSAVE(I)=S
      WRITE(6,8)S
    8 FORMAT('0','STD. ERROR OF ESTIMATE=  ',F13.6//)
C
C  CALCULATE THE COVARIANCE MATRIX OF THE ESTIMATES
      CALL ARRAY(1,NNI,NNI,4,4,XTX,XTX)
      DO 141 J=1,NNI
      DO 142 K=1,NNI
      XTX(J,K)=S1*XTX(J,K)
  142 CONTINUE
  141 CONTINUE
      WRITE(6,5050)
 5050 FORMAT('0',T10,'COVARIANCE MATRIX OF THE ESTIMATES'/)
      DO 143 J=1,NNI
      WRITE(6,144)(XTX(J,K),K=1,NNI)
  143 CONTINUE
  144 FORMAT(2X,8E13.5)
C
C  CALCULATE ADJUSTED MUNTIPLE CORRELATION COEFICIENT
      SUM=0.0
      DO 150 J=1,N
      SUM=SUM+Y(J)
  150 CONTINUE
      AN=N
      YBAR=SUM/AN
      DO 160 J=1,N
      Y(J)=Y(J)-YBAR
  160 CONTINUE
      CALL GTPRD(Y,Y,YY,N,1,1)
      CN=N-1
      R=1.0-(S1/(YY(1)/CN))
      WRITE(6,9)R
    9 FORMAT('0','ADJUSTED MULTIPLE CORRELATION CCEFICIENT',F9.6)
C
C  CONPUTE LEAST SQUARES (INDEPENDENT) PREDICTION INTERVAL
      CALL ARRAY(2,NNI,NNI,4,4,XTX,XTX)
      CALL ARRAY(1,N,NNI,36,4,X,X)
```

```
      DO 5100 K=1,N
      DO 5000 J=1,NNI
  990 XOBS(J)=X(K,J)
 5000 CONTINUE
      WRITE(6,5700)(XOBS(IK),IK=1,NNI)
 5700 FORMAT(' ',4F13.3)
      CALL GTPRD(XOBS,B,YHAT,NNI,1,1)
      TCSAVE(K)=TCSAVE(K)+YHAT(1)
      CALL GMPRD(XTX,XOBS,PSAVE,NNI,NNI,1)
      CALL GTPRD(XOBS,PSAVE,PHAT,NNI,1,1)
  991 ASAVE=S1+PHAT(1)
  992 A(K)=A(K)+ASAVE
 5100 CONTINUE
  300 CONTINUE
      DO 888 I=1,N
      A(I)=1.96*SQRT(A(I))
  888 CONTINUE
      DO 5200 L=1,N
      PIM(L)=TCSAVE(L)-A(L)
      PIP(L)=TCSAVE(L)+A(L)
 5200 CONTINUE
      WRITE(6,9005)
 9005 FORMAT('1'//////////16X,'TABLE OF LS (INDEPEND) PREDICTION INTERVA
     1L VALUES'/'0',17X,'PI +',9X,'PREDICTED',4X,'ACTUAL',7X,'PI -'/'0')
      DO 5300 J=1,N
      WRITE(6,5400)J,PIP(J),TCSAVE(J),DATA(J,36),PIM(J)
 5300 CONTINUE
 5400 FORMAT(' ',11X,I2,4F13.6)
      STOP
      END
```

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CERs	

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This study addresses the problem of uncertainty in the aggregation of subsystem cost estimating relationships (CERs) for destroyer type naval ships. The case of correlations between subsystems is discussed with respect to total cost estimates, total cost variance, and prediction intervals. The joint generalized least squares method (JGLS) is developed and applied

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to the ship cost estimating problem. The models analyzed were developed by D. M. Hernon and R. R. McCumber, Estimation of Destroyer Type Naval Ship Procurement Costs, utilizing the data base from Resource Management Corporation Report CR-058, The Navships Cost Model. One model utilizes 9 subsystem CERs to obtain a total cost estimate, while the second model utilizes 4 subsystem CERs. Three solution methods were used for purposes of comparison; the JGLS method assuming correlated subsystems, the least squares method assuming correlated subsystems, and the least squares method assuming independent subsystems. The JGLS method is shown to provide the most meaningful results for correlated subsystems. A computer program and user's guide are provided to conduct JGLS analysis.

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